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REASONS FOR GRAVITATIONAL MASS AND THE PROBLEM OF QUANTUM GRAVITY

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Abstract

The problem of quantum gravity is treated from a radically new viewpoint based on a detailed mathematical analysis of what the constitution of physical space is, which has been curried out by Michel Bounias and the author. The approach allows the introduction of the notion of mass as a local deformation of space regarded as a tessellation lattice of founding elements, topological balls, whose size is estimated as the Planck one. The interaction of a moving particle-like deformation with the surrounding lattice of space involves a fractal decomposition process that supports the existence and properties of previously postulated *inerton clouds* as associated to particles. The cloud of inertons surrounding the particle spreads out to a range $\Lambda = \lambda c / \upsilon$ from the particle where υ and c are velocities of the particle and light, respectively, and λ is the de Broglie wavelength of the particle. Thus the particle's inertons return the real sense to the wave ψ-function as the field of inertia of the moving particle. Since inertons transfer fragments of the particle mass, they play also the role of carriers of gravitational properties of the particle. The submicroscopic concept has been verified experimentally, though so far in microscopic and intermediate ranges.

1. Introduction

Gravity was initially constructed as a classical theory. However, the discovery of quantum phenomena allowed researchers to assume the existence of a possible quantum origin for gravity. This established a new discipline known today as quantum gravity. Nowadays one can distinguish several major branches in the study of quantum gravity. Newer approaches abandon the early idea of general relativity as a dynamical system and settle up quantum field theories exhibiting general covariance and/or manifolds. Concepts developed in particle physics, which investigate the possibilities of propagation of quantum fields with the specification of their internal structure, such as gauge group, fields, and interactions, attract a wider interest. Significant role places symmetries of quantum field theories, which reflect the mappings of manifolds. Thus quantum gravity is related to manifold topology. Good reviews on modern approaches to the description of quantum gravity, such as quantization of gravity, the non-renormalisability of gravity, supergravity, superstring theory (or M-theory), loop quantum gravity, gauge theories and others, one can find, e.g. in Refs. [1-4]; a review of the latest trends in the study of quantum space-time is available in Balachandran [5].

Sarfatti [6,7] has sketched very recent progress in the foundations of quantum theory representing his views on Einstein's gravity including both dark energy and dark matter, which emerge macro-quantum phenomena in the sense of Penrose's "off diagonal long range order" inside the vacuum. This is in line with Anderson's "more is different" and Sakharov's "metric elasticity" approaches. His result is a background-independent non-perturbative model that agrees with loop quantum gravity. He offers a model for the formation of the vacuum condensate "inflation scalar field" filled with bound virtual electron-positron pairs; these pairs force the globally flat quantum electro-dynamic vacuum to be unstable. By Sarfatti, Einstein's c-number space-time emerges as the more stable vacuum (quite similar to the superconducting vacuum state for a condensate of real electron pairs where the attractive virtual phonons overpower the repulsive virtual photons in the real electron-electron pair coupling). Sarfatti's means are de Broglie-Bohm's pilot wave interpretation of quantum theory and the usual "two fluid model" in which the residual random zero-point vacuum fluctuations of all available dynamical fields are the "normal fluid". Thus Sarfatti reasonably concludes: "There is no such thing as a local classical world. We live in a local macro-quantum world!" His results [6,7] are close to Kleinert's and Zaanen's [8] who have developed the 4D "world liquid crystal lattice" model. This model of Einstein's gravity is consistent with that of a foam loop quantum gravity. By Kleinert and Zaanen, general relativity in which the perfect world crystal lattice with unit cells at the Planck scale corresponds to globally flat space-time with zero curvature in all components not only space curvature. In this model, Einstein's curvature corresponds to a disclination defect density in the long wave limit of the elastic world crystal distortion.

Thus gravity is still considered different from the other physical forces, whose classical description involves fields (e.g. electric or magnetic fields) and field carriers propagating in space-time. General relativity says that the gravitational force is related to the curvature of space-time itself, i.e. to its geometry. We can see that in gravitational physics space-time is treated as a dynamical field, though in non-gravitational physics space-time emerges as the scene on which all physical processes take place. Therefore available approaches to quantum gravity represent a collection of liable geometries of space-time. It is pity that such interesting models of space-time and quantum gravity seems to remain metaphysical by its nature by having no contact with experiments in principle.

Toh [3] said, after C. J. Isham, that rather than quantising gravity, one should seek a quantum theory that would yield general relativity as its classical limit. He then adds that the main obstruction here is the lack of a starting point to construct such a quantum theory. A radically new theory may require the re-examination, from their foundation, the concepts of space, time, and matter [9].

Santilli [10] justly notes that the exterior gravitational field of a mass originates entirely from the total energy-momentum tensor of the electromagnetic field of all elementary constituents of this mass and hence seems gravitation has to be a mere additional manifestation of electromagnetism. And he also mentions together with many other researchers that Einstein's gravitation explains the $43''$ of the precession of the perihelion of Mercury, but cannot explain the basic Newtonian contribution!

In the present paper we give a demonstration of an alternative viewpoint on quantum gravity, in fact a tenable starting point for a new quantum theory, which, in contrast to conventional approaches, allows the unification of gravity and electromagnetism (on the basis of the fractal origin of mass and charge, though such unification requires separate studies) and the direct test experimentation. Experimental aspects and the verification of the theory presented receive a detailed consideration.

2. Preliminaries

2.1. Constitution of physical space and the foundations of space-time

Physical mathematics developed in the 20^{th} century is still treated as an effective method for theoretical studies in particle and gravitational physics. Various ill-determined fields, such as the wave ψ - function and spinor $\overline{\psi}$ and more abstract, their gauge transformations, abstract entities (strings, loops, and etc.), various kinds of symmetries and so on, are those host elements of modern theoretical constructions, which nowadays are used in mathematical modelling of space-time and fundamental interactions.

Having developed an alternative concept of quantum gravity, we have to elaborate a new approach to space-time. Starting from pure mathematics, namely, set theory, topology and fractal geometry, in which any element is strictly defined, Bounias and the author [11-14] have shown that a physical space can exist as a collection of closed topologies in the intersections of abstract topological subspaces provided with non-equal dimensions.

It seems that Nottale [15] was the first who applied fractal geometry to the description of space-time. He studied relativity in terms of fractality basing his research on the Mandelbrot's concept of fractal geometry. Nottale introduced a scale-relativity formalism, which allowed him to propose a special quantization of the universe. In his theory, scale-relativity is derived from a special application of fractals. In his approach, the fractal dimension D is defined from the variation with resolution of the main fractal variable, i.e. the length l of fractal curve plays a role of a fractal curvilinear coordinate. Nottale's approach leads to the conclusion that a trajectory of any physical system diverges due to the inner stochastic nature that is caused by the fractal laws. Again, in this approach fractality is associated with the length of a curve as such.

In our works [11-14] we have shown that fractal geometry can be derived completely from other mathematical principles, which becomes possible on the basis of reconsidered fundamental notions of space, measure, and length; this allowed us to introduce deeper first principles for the foundations of fractal geometry.

First of all we argue [11-14] that since in mathematics some set does exist, a weaker form can be reduced to the existence of the empty set Ø. An abstract lattice of empty set cells Ø has been shown to be able to account for a primary substrate in a physical space. Providing the empty set, with special combination rules including the property of complementarity, allowed us to define a magma. The magma $\varnothing^{\varnothing} = \{\mathbf{C}, \varnothing\}$ constructed with the empty hyperset together with the axiom of availability form a fractal tessellation lattice. Space-time is represented by ordered sequences of topologically closed Poincaré sections of this primary space. We have demonstrated that the antifounded properties of the empty set provide existence to a lattice involving a tessellation of the corresponding abstract space with empty topological balls. The founding element, i.e. a topological ball, or a primary cell, of this mathematical lattice called the *tessel-lattice*, can be estimated at the size close to the Planck one, or more exactly, ~ 10^{-35} m. Discrete properties of the tessellattice allow the prediction of scales at which microscopic to cosmic structures should occur.

Deformations of primary cells by exchanges of empty set cells allow a cell to be mapped into an image cell in the next section as far as mapped cells remain homeomorphic. However, if a deformation involves a fractal transformation to objects, there occurs a change in the dimension of the cell and the homeomorphism is not conserved. Then the fractal kernel stands for a *particle* and the reduction of its volume (together with an increase of its area up to infinity) is compensated by morphic changes of a finite number of surrounding cells. Therefore, the state described is associated with the availability of an actual *mass* in the tessellattice, i.e. a local deformation of the tessellattice (a deformation of a cell) is identified with what we call “mass” in conventional physics. The interaction of a moving particle-like deformation with the surrounding lattice involves a fractal decomposition process that supports the existence and properties of previously introduced *inerton clouds* [16,17] as associated to particles.

Thus the tessellattice is found in the degenerate state that does not manifest itself as matter; cells of the tessellattice are empty sets and this is a typical vacuum with its flat geometry. However, the tessellattice emerges when a deformed cell (i.e. a massive particle) starts to move: the decomposition of this local moving deformation entrains other cells of the tessellattice. Such a decomposition, or (in the physics language) clouds of inertons, should allow the experimental verification. We will come back to this issue below.

The existence of closed topological structures gives the intersection of two spaces having nonequal dimensions owns its accumulation points and is therefore closed. In other words, the intersection of two connected spaces with nonequal dimensions is topologically closed. This allowed the representation of the fundamental metrics of our space-time by a convolution product where the embedding part

$$U4 = \int \left(\int_{dS} dx\, dy\, dz \right) * d\psi(w) \tag{1}$$

where dS is the element of space-time and $d\psi(w)$ the function accounting for the extension of 3-D coordinates to the 4^{th} dimension through convolution ($*$) with the volume of space.

Time is an emergent parameter indexed on non-linear topological structures guaranteed by discrete sets. This means that the foundation of the concept of time is the existence of order relations in the sets of functions available in intersecting sections. The symmetric difference between sets and its norm can be treated as a new, more general, *non-metric distance*. As the set distance *D* is the complementary of objects, the system stands as a manifold of open and closed subparts. Mapping of these manifolds from one to another section, which preserve the topology, represents a reference frame in which topology has allowed one to specify the changes in the configuration of main components: if morphisms are observed, then this enables the interpretation as a motion-like phenomenon, when one compares the state of a section with the state of mapped section. Hence a space-time-like sequence of Poincaré sections is a non-linear convolution of morphisms. This means that time is not a primary parameter. And the physical universe has no longer any beginning: time is just related to ordered perceptions of existence, not to existence itself. The topological space does not require any fundamental difference between reversible and steady state phenomena, nor between reversible and irreversible process. Rather relation orders simply hold on non-linearity distributed topologies, and from rough to finest topologies.

The mapping of two Poincaré sections is assessed by using a natural metrics of topological spaces. Let $\Delta(A, B, C, ...)$ be the generalized set distance as the extended symmetric difference of a family of closed spaces,

$$\Delta(A_i)_{i \in N} = \mathbf{C}_{\cup\{A_i\}} \cup_{i \neq j} (A_i \cap A_j). \tag{2}$$

The complementary of Δ in a closed space is closed. It is also closed even if it involves open components with nonequal dimensions. In this system $m\langle(A_i)\rangle = \Delta$ has been associated with the instant, i.e., the state of objects in a timeless Poincaré section. Since distances Δ are the complementaries of objects, the system stands as a manifold of open and closed subparts.

The set-distance provides a set with the finer topology and the set-distance of nonidentical parts provides a set with an ultrafilter. Regarding a topology or a filter founded on any additional property ($\perp$) this property is not necessarily provided to a Δ-filter. The topology and filter induced by Δ are thus respectively the finer topology and an ultrafilter. The mappings of both distances and instants from one to another section can be described by a function called the *moment of junction*, since it has the global structure of a momentum [12]. The moment of junction allowed us to investigate the composition of indicative functions of the position of points within the topological structures and to account for elements of the differential geometry of space-time. Therefore, the moment of junction enables the formalization of the topological characteristics of what is called *motion* in a physical universe. It is the motion that has been called by de Broglie as the major characteristic determining physics. While an identity mapping denotes an absence of motion, that is a null interval of time, a nonempty moment of junction stands for the minimal of any time interval. Sidharth [18] tried to argue that a minimum space-time interval should exist and that “one cannot go to arbitrarily small spacetime intervals or points”. In our sense, there is not such a “point” at all: only instants [11,12] that at bottom of fact do not reflect timely features.

Space-time versus neo-Lorentzian interpretation of relativity is conciliated in the tessellation space. Indeed, according to Bouligand, Minkowski, Hausdorff and Besicovich when all intervals (at least nearly) have the same size, then the various dimension approaches are reflected in the resulting relation [12]

$$n \cdot (\rho)^{e} \cong 1 . \tag{3}$$

Here e is the Bouligand similarity (integer) exponent that specifies a topological dimension and this means that a fundamental space E can be tessellated with an entire number of identical topological balls exhibiting a similarity with E, upon the similarity coefficient ρ; n is the number of parts.

The fractality of particle-giving deformations gathers its spatial parameters and velocities into a self-similarity expression (on the basis of relation (3)), which provides the space-to-time connection required by special relativity.

Indeed, the fractality of particle-giving deformations gathers its spatial parameters φ_{i} and velocities υ into a self-similarity expression that provides a space-to-time connection. Indeed, let φ_0 and υ_0 be the reference values; then the similarity ratios are $\rho(\varphi) = (\varphi_{\mathrm{i}} / \varphi_0)$ and $\rho(\upsilon) = (\upsilon / \upsilon_0)$. Therefore,

$$\rho(\varphi)^{\mathrm{e}} + \rho(\upsilon)^{\mathrm{e}} = 1. \tag{4}$$

The left-hand side of Eq. (4) includes only space and space-time parameters, because $(\varphi_{\mathrm{i}})=\{$distances ($l$) and masses ($m$)$\}$. Hence we can set $m_0/m = l/l_0$, so that

$$(m_0 / m)^{\mathrm{e}} + (\upsilon / \upsilon_0)^{\mathrm{e}} = (l_0 / l)^{\mathrm{e}} + (\upsilon / \upsilon_0)^{\mathrm{e}} = 1. \tag{5}$$

The geometry at $\mathrm{e}>1$ escapes the usual space-time; the necessity of an embedding 4-D timeless space (1) ensures the coefficient e to be equal to 2 (see Ref. [12] for details). Then the boundary conditions give rise to the following results for distances l, l_0 and masses m, m_0:

$$(m_0 / m)^2 + (\upsilon / \upsilon_0)^2 = 1 \quad \Leftrightarrow \quad m = m_0 / \sqrt{1 - (\upsilon / \upsilon_0)^2}\ ; \tag{6}$$

$$(l / l_0)^2 + (\upsilon / \upsilon_0)^2 = 1 \quad \Leftrightarrow \quad l = l_0 \sqrt{1 - (\upsilon / \upsilon_0)^2}\ . \tag{7}$$

The Lagrangian L should obey a similar ratio (5), such that

$$(L / L_0)^2 + (\upsilon / \upsilon_0)^2 = 1. \tag{8}$$

If we take $L_0 = -m\upsilon_0^2$, we obtain

$$L_0 = -m\upsilon_0^2 \sqrt{1 - (\upsilon / \upsilon_0)^2}\ . \tag{9}$$

By analogy with special relativity, m, υ and L are the parameters of a moving object and υ_0 is the speed of light.

To summarize, topology, set theory and fractal geometry allow the construction of the new theory of physical space [12-14], which in fact emerges as a mathematical lattice:

$$\mathrm{F(U)} \cup \mathrm{(W)} \cup \mathrm{(c)} \tag{10}$$

where (c) is the set with neither members nor parts, accounts for relativistic space and quantic void, because (i) the concept of distance and the concept of time have been defined on it and (ii) this space holds for a quantum void since it provides a discrete topology, with quantum scales and it contains no "solid" object that would stand for a given provision of physical matter. The sequence of mappings of one into another structure of reference (e.g. elementary cells) represents an oscillation of any cell volume along the arrow of physical time. This rigorously supports submicroscopic (quantum) mechanics constructed by the authors in a series of recent works [16,17,19-21].

2.2. Principle of equivalence

Very recently de Haas [22] studying paradoxes in the early writings (the first third of the 20th century) on the relativity and quantum physics has revealed that a combination of the Gustav Mie's [23] theory of gravity and the Louis de Broglie's [24] harmony of phases of a moving particle results in the principle of equivalence for quantum gravity. De Haas notes that Mie's findings on the description of gravity have been very important and have been used by Gilbert to transform the Mie's Hamiltonian variation principle into a general covariant variation principle, which then has been used by other physicists. De Haas considering Mie's variational principle has shown that the Mie-de Broglie version of the Hamilton variational principle directly holds in the quantum domain,

$$\delta\int H\,d\tau \;\propto\; \hbar\,, \tag{11}$$

though in the classical limit $\hbar \to 0$ one gets a conventional non-quantum version of the Hamiltonian principle, $\delta\int H\,d\tau \;= 0$. Thus the Mie-de Broglie theory of quantum gravity analyzed by de Haas accounts for a principle of equivalence of gravitational m_{gr} and inertial m_{in} masses. Namely, the equality $m_{\mathrm{gr}} = m_{\mathrm{in}}$, which is held in a rest-frame of the particle in question, becomes invalid in a moving reference frame. In the quantum context, this equality should be transformed to the principle of equivalence of the appropriate phases, $\varphi_{\mathrm{gr}} = \varphi_{\mathrm{in}}$. This relation ties up the gravitational and inertial energies of the particle and also shows that the gravitational mass is completely allocated in the inertial wave that guides the particle. Mie's original result for the gravitational mass was (see sections 42 to 45 in Ref. [23])

$$m_{\mathrm{gr}} = m_0 \sqrt{1 - \upsilon^2 / c^2}\,, \tag{12}$$

though the inertial mass was identified with the total mass, $m_{\mathrm{in}} = m_0 / \sqrt{1 - \upsilon^2 / c^2}$. Thus the gravitation as such seems in fact is a pure dynamic phenomenon, which on the quantum level has to be associated with the matter waves.

Consequently, de Haas [22] has revived de Broglie's initial physical interpretation of wave/quantum mechanics, i.e. the pilot wave interpretation of de Broglie, and also introduced a fresh wind into the problem of quantum gravity. By de Haas, it turns out that a particle moving through the space deforms the metrics on a quantum local scale in such a way that the inertial energy flow $E_{\mathrm{in}} \mathbf{v}_{\mathrm{group}}$ becomes concentrated in the particle's wave packet, though the gravitational energy flow $E_{\mathrm{gr}} \mathbf{v}_{\mathrm{particle}}$ becomes dislocated in it. Nevertheless, on a macroscopic scale, the equality between these two kinds of energy is preserved of course. The result of de Haas sounds in fact very realistic, because he could easily link Mie's source of gravitational energy to the trace of the inertial stress-energy tensor that takes the role of the source of gravitational energy in modern concepts.

It is interesting to note the de Haas' finding, i.e. that the principle of equivalence of appropriate phases of inertial and gravitational energies, on both ontological and mathematical levels strongly supports the author's own researches [16,17,19-21] on submicroscopic mechanics of particles moving in the tessellation space. For the major part, this is the interpretation of the matter waves as a complicated system that consists of the particle and its inerton cloud, which move in the space tessel-lattice periodically exchanging energy and momentum due to the strong interaction with the tessel-lattice's cells.

3. The notion of mass

A particulate ball (i.e. a deformed cell of the tessel-lattice), as indicated above [11-14], provides formalism describing the elementary particles. In this respect, mass is represented by a fractal reduction of volume of a topological ball (or, in other words, a cell of the tessel-lattice), while just a reduction of volume as in degenerate cells is not sufficient to provide mass, because a dimensional increase is a necessary condition. The mass m_A of a particulate ball A is a function of the fractal-related decrease of the volume of the ball,

$$m_\mathrm{A} \propto \frac{V_{\text{degenerate topol. ball}}}{V_{\text{particulate ball}}} \left(\mathrm{e}_\kappa - 1\right)_{\mathrm{e}_\kappa \geq 1} \tag{13}$$

where e is the Bouligand exponent, and $(\mathrm{e}_\kappa - 1)$ the gain in dimensionality given by the fractal iteration; the index κ denotes the possible fractal concavities affecting the particulate ball.

In condensed matter physics, any foreign particle introduced in the lattice leads to its deformation that extends from several to many lattice constants. Such a deformation is called the *deformation coat* in solids or the solvation in liquids. This takes place within an ordered/disordered lattice and the network of molecules. Therefore, such a deformation is typical for any kind of a lattice and seems there are no reasons to except the tessel-lattice from this rule.

In that way a particulate cell in the tessellattice has to generate its own deformation coat; it has been argued [16,17,19-21] that the radius of the coat coincides with the Compton wavelength $\lambda_{\text{Com}} = h/mc$ of the particle under consideration. In the deformation coat all cells are found in the massive state, though beyond the coat the tessellattice's cells are still massless. That is why this deformation coat can be called a *crystallite* in which its entities (i.e. massive cells) undergo oscillations, as is the case with the conventional crystal lattice. The total mass of the crystallite's cells is equal to the mass of the particle [19].

A moving particle drags the crystallite all over the place: The resultant readjustment of cells from the massless to massive state and again to the massless one occurs along the velocity vector **v** of the particle. In transversal directions the state of superparticles remains practically unaltered: cells surrounding the particle continuously store the same massive state and hence in these directions cells are rather rigid. That is why it is reasonable to assume that cells in the crystallite vibrate in transversal vibrations. These transversal vibrations of the crystallite can be called the (transversal) oscillating mode [25]. The energy of the oscillating mode of the crystallite is $E = \hbar\omega$ where $E = m_0 c^2 / \sqrt{1-\upsilon^2/c^2}$ is the total energy of the moving particle; in the immobile state $E \to E_0 = \hbar\omega_0 = m_0 c^2$ [19].

4. Motion of a particle and the field of inertia

The motion of a particle, of course, must involve the interaction with the ambient space. A detailed study of this motion, which results in submicroscopic mechanics, is described in Refs. [16,17,19-21]. The appropriate Lagrangian, which contains terms describing the particle, its interaction with the tessellattice and the cloud of spatial excitations generated at collisions of the moving particle with the tessel-lattice, can be written in the form [17]

$$L = -m_0 c^2 \left\{ 1 - \frac{1}{m_0 c^2} \left[m_0 \dot{x}^2 - \frac{2\pi}{T} \sqrt{m_0 \mu_0} \left(x\dot{\chi} + \upsilon\chi \right) + \mu_0 \dot{\chi}^2 \right] \right\}^{1/2}; \quad (14)$$

here m_0, x and $\dot{x}$ are the mass, the position and the velocity of the particle at a moment *t*, where time *t* is considered as a natural parameter; μ_0, χ and $\dot{\chi}$ are the mass, the position and the velocity of centre-of-mass of the cloud of spatial excitations called *inertons*; *T* is the period of collisions between the particle and its inerton cloud; υ is the initial velocity of the particle; *c* is the velocity of inertons, which may exceeds the speed of light. The value of the Lagrangian (14) coincides with that given formally by expression (9).

The Euler-Lagrange equations constructed for variables $q, \dot{q} = \{x, \dot{x};\ \chi, \dot{\chi}\}$, i.e.

$$d/dt\,(\partial L/\partial \dot{q}) - \partial L/\partial q = 0 \quad (15)$$

yield the following continuous solutions

$$x(t) = \upsilon t + (\upsilon T/\pi)\left\{(-1)^{[t/T]}\cos(\pi t/T) - \left(1 + 2[t/T]\right)\right\}, \tag{16}$$

$$\dot{x}(t) = \upsilon \cdot \left(1 - \left|\sin(\pi t/T)\right|\right), \tag{17}$$

$$\chi(t) = (\Lambda/\pi)\left|\sin(\pi t/T)\right|, \tag{18}$$

$$\dot{\chi}(t) = (-1)^{[t/T]}\, c\cos(\pi t/T) \tag{19}$$

where the notation $[t/T]$ means an integral part of the integer $t/T.$ Besides, in expressions (16) to (19),

$$\upsilon T = \lambda \quad \text{and} \quad cT = \Lambda. \tag{20}$$

It is readily seen that parameters λ and Λ play the role of free pass lengths of the particle and the inerton cloud, respectively. The solutions (16) to (19) show that the moving particle periodically exchanges the velocity (hence the kinetic energy and the momentum) with its inerton cloud. That is, the particle emits inertons within each odd sections $\lambda/2$ of the particle path and its velocity gradually decreases from υ to 0; then the particle absorbs inertons within even sections $\lambda/2$ of its path and the particle velocity progressively increases from 0 to υ. Besides, the following relationship comes out from expressions (20),

$$\Lambda = \lambda c/\upsilon, \tag{21}$$

which connects the particle’s and the inerton cloud’s free path lengths.

Orthodox quantum mechanics emerges from the canonical Hamiltonian obtained from the Lagrangian (14) if we pass to the Hamilton-Jacobi formalism [16,17,19]. Such a transition allows us to derive de Broglie’s relationships

$$E = h\nu \quad \text{and} \quad \lambda = h/m\upsilon. \tag{22}$$

Thus submicroscopic mechanics shows that the de Broglie wavelength is nothing else as the free path length, or the spatial amplitude, λ of the particle. The free path length of the inerton cloud, or the amplitude of the inerton cloud, Λ (21) characterises the distance to which the particle’s inertons spread from the particle. E from de Broglie's relationships (22) has been determined as $m\upsilon^2/2$, the mass m is the total mass, which is also the inertial mass, $m = m_0/\sqrt{1-\upsilon^2/c^2}$, that agrees with Mie's result [23]; ν is linked with the period of collisions T, namely, $\nu = 1/2T$. Planck's constant h gets an interpretation of the minimum increment of the particle action in the cyclic process, i.e. the motion with the periodic exchange of energy between the particle and the accompanied cloud of inertons, which is only guided by the space tessel-lattice.

The availability of relationships (22) allows one immediately to derive the Schrödinger equation [26], Lorentz-invariant in our case due to the invariance of time, i.e. proper time, entered the equation. Then the wave ψ-function, which so far has been treated as purely abstract, receives the rigorous physical interpretation of the field of inertia, or the inertons field, surrounding the moving particle in the range covered by the amplitude Λ. This means that inertons accompanying a moving particle represent a substructure of the particle's matter waves.

5. The contraction of mass

The behavior of a moving canonical particle pulling its deformation coat and surrounded by the cloud of inertons can be studied also in the framework of a hydrodynamic approach. In hydrodynamics, the notion of point particle is limited by the proper size of the considered element of the liquid. This size is enormous as compared with the particle size. The motion of such an element in the space tessel-lattice treated as a world fluid can be described by the basic equation of hydrodynamics

$$\rho\, d\mathbf{v}/dt = -\nabla P \qquad (23)$$

where ρ is the liquid element density and P is the pressure of the liquid on the moving element. We assume that the motion is adiabatic when the change in pressure on the side of the liquid upon the element is proportional to the variation in density of this element and then

$$(\partial P/\partial \rho)_{\text{entropy}} = c^2 \qquad (24)$$

where c is the maximum velocity for this liquid, i.e. sound velocity.

Equation (23) is non-linear and consequently allows for multiple solutions. However, there is only one possibility when this equation becomes linear, and, therefore, there is a single solution. That situation is realized when we examine the motion of an element at the limits of tolerance, i.e. when space-time derivatives become discrete. Note that recently Dubois [27,28] has successfully applied space-time derivatives deduced from forward and backward derivatives in computation for the study of quantum relativistic systems and electromagnetism. By the way, Dubois shows that [28] the masses of particles could be interpreted as properties of space-time shifts, which is also in support of our definition of mass as a local deformation of space.

The non-stationary motion of the element of our liquid allows the representation of the equation of motion (23) in a discrete form [17]. At the discrete consideration the substantial derivative has to be transformed as follows

$$\frac{d\,f(s)}{ds} = \lim_{\Delta s \to \Delta s_0} \frac{f(s+\Delta\, s) - f(s)}{\Delta\, s} = \lim_{\Delta s \to \Delta s_0} \frac{\Delta f}{\Delta\, s} \tag{25}$$

where Δs_0 stands for the size of the element Δl_0 or the time interval Δt_0 needed to pass the section equal to the element's size. The submicroscopic consideration says that the smallest size of the macroscopic element along the element path has to be restricted by the perturbation of space caused by the inerton cloud; therefore, $\Delta l_0 = \lambda / 2$ and $\Delta t_0 = T / 2$. Let υ and ρ_0 be the initial values of the velocity and the density of the element in question, respectively. Then Eqs. (23) and (24) can be written as

$$\rho \frac{\Delta \upsilon}{T/2} = -\frac{\Delta P}{\lambda / 2}, \tag{26}$$

$$\Delta P / \Delta \rho = c^2 . \tag{27}$$

Eqs. (26) and (27) result in equation

$$\rho\, \Delta \upsilon\, \lambda / T = -c^2 \Delta \rho . \tag{28}$$

In Eq. (28) $\lambda / T = \upsilon$ (see expression (20)). Since by definition $\Delta f = f_{\text{current}} - f_0$ where f_0 stands for the initial value of the function, we obtain $\Delta\upsilon = -\upsilon$ for odd sections $\lambda/2$ of the element path where the element velocity decreases to zero. When the speed of the element changes from the maximum to the minimum magnitude, the pressure, on the contrary, changes from the minimum (P) to the maximum (P_0) magnitude and therefore $\Delta P = P - P_0$. Then from Eq. (27) we get $\Delta\rho = \rho - \rho_0$. Substituting the corresponding values to equation (28) we arrive at the expression

$$\rho = \rho_0 /(1 - \upsilon^2 / c^2). \qquad (29)$$

In the case of even sections $\lambda/2$ of the element path where the element velocity increases from zero to υ, we get $\Delta\upsilon = \upsilon$ and then $\Delta\rho = \rho_0 - \rho$. These values of the parameters transform Eq. (28) to the same expression (29).

From definition (13), the mass is inversely proportional to the volume, which it occupies in the tessel-lattice, we may write for the mass of the liquid element $m_0 \propto 1/V_0$. Therefore, $\rho_0 = m_0 / V_0 \propto (1/V_0)^2$, and we derive from expression (29)

$$m = m_0 / \sqrt{1 - \upsilon^2 / c^2}. \qquad (30)$$

This result is in line with that (6) formally obtained by using the fractality of particle-giving deformations.

Thus the model correctly describes the increase of mass and reduction of dimensions of a moving object along a path and in this respect the model complies with the Lorentz hypothesis on the change of the geometry of a particle in motion.

6. Gravitational potential

What is a mechanism of the emission of inertons from a moving particle? What is the reason to turn emitted inertons back to the particle? What do inertons transfer exactly?

To answer these questions we first of all should mention that a gentle hint to the appropriate mechanism was given by de Broglie [29,30]. De Broglie indicated that the corpuscle dynamics was the basis for the wave mechanics. With the variational principle, he obtained and studied the equations of motion of a massive point reasoning from the typical Lagrangian (9). The study showed that the dynamics had the characteristics of the dynamics of the particles with a variable proper mass. Nevertheless, in de Broglie's research the velocity υ of the point still was constant along a path.

Owing to the availability of the crystallite surrounding the particle in the tessel-lattice, we should examine the interaction of the particle with the crystallite's oscillating mode. An appropriate mechanism has been described in Ref. [25] in detail. The main aspects of the particle – oscillating mode interaction are as follows. At each collision of the moving particle with the mode, whose energy is supported by the ambient tessellattice, the particle loses a fragment δV of its total deformation $V_{\rm part}\sqrt{1-\upsilon^2/c^2}$ (in line with the fractal decomposition principle [12]) or, in other words, the inertial mass (30) decreases on a value of the inerton mass μ_1. The created quasi-particle, i.e. inerton, is characterized by the kinetic energy and the momentum obtained from the mode and the particle, which enables the inerton to go off the particle.

Inertons are emitted from the particle until the particle is moving. Since the value of the particle velocity oscillates along the particle path, $\upsilon \to 0$ in each odd section $\lambda/2$ and $0 \to \upsilon$ in each even section $\lambda/2$, the decay of the total mass of the particle, called the relativistic mass, (30) must follow the behavior of the particle. Namely, the total particle mass has also to oscillate along the particle's path: $m \to 0$ and then $0 \to m$. The total particle mass (30) comes to the inerton cloud that spreads over the space tessellattice up to a distance $r = \Lambda$.

A number of inertons created at the decay of the particle mass in each odd section $\lambda/2$ is huge and can be evaluated by the number N of collisions of the particle with tessel-lattice's cells in this section, i.e. $N \sim \lambda/l_{\rm Planck}$ (recall $l_{\rm Planck}$ $\sim 10^{-35}$ m is the size of a cell); for instance, if $\lambda = 0.1$ nm, the number of inertons created by the particle is on the order of $N = \lambda/l_{\rm Planck} \sim 10^{26}$. The value of the mass μ_1 of an inerton can widely varies and by the assessment [30] $\mu_1 \approx 10^{-85}$ to 10^{-45} kg.

Emitted inertons should come back to the particle bringing fragments of its deformations in each section $\lambda/2$ of the particle path. In other words, absorbed inertons should restore the initial state of the moving particle, i.e. its mass and velocity. The simplest mechanism of such behaviour of inertons, which strings them to the particle, can be understood only in the framework of a *general dynamics of the tessel-lattice*.

Having returned back to the particle, the inerton in question has initially to come to a stop. The resetting of the inerton means that this is the elastic tessellattice that guides it at a distance Λ from the particle and then replaces again. Therefore, the emitted inerton should undergo an elastic influence on the side of the tessellattice. Hence we have to supplement the Lagrangian (14) by additional terms that will lead to the Euler-Lagrange equations describing changes in both an elastic property of the inerton and that of the tessellattice. What may vary in the bound inerton? Obviously the degree of its deformation may only vary, i.e. the value of the inerton mass. In other words, this local deformation migrating from cell to cell slowly drops while inducing a *rugosity* in ongoing cells, which in physics terms is the *tension of the tessel-lattice* if applied to the cloud of inertons as a whole. The induction of the rugosity, or tension, does not destroy the morphism of cells, but introduces a tension to fractal deformations, which can slightly translate the cells from their equilibrium positions in the tessellattice. The tension is removed by the energy stored in the tessel-lattice, or in other words, the tessel-lattice restores its initial state in which every cell occupies its own equilibrium position.

Thus the Lagrangian (14) is extended to

$$L = -m_0 c^2 \left\{ 1 - \frac{1}{m_0 c^2} \left[m_0 \dot{x}^2 - \frac{2\pi}{T} \sqrt{m_0 \mu_0} \left(x\dot{\chi} + \upsilon \chi \right) + \mu_0 \dot{\chi}^2 \right] \right. \\ \left. + \left[\frac{T^2}{2m_0^2} \dot{m}^2 + \frac{T^2}{2\Lambda^2} \dot{\vec{\xi}}^2 - \frac{T}{m_0} \dot{m} \nabla \vec{\xi} \right] \right\}^{1/2} . \tag{31}$$

Here $m = m(\vec{r}, t)$ is the current mass of the {particle + inerton cloud}-system; $\vec{\xi} = \vec{\xi}(\vec{r}, t)$ is the current value of the rugosity of the tessellattice in the range covered by the system. Geometrically the rugosity $\vec{\xi}$ depicts the state in which the tessel-lattice cells covered by the inertons cloud do not have any volumetric deformation, as the cells here are massless. However, they become shifted from equilibrium positions in the tessel-lattice along the particle vector velocity $\vec{\upsilon}$.

Proceeding to the Euler-Lagrange equations for variables m and $\vec{\xi}$, we have to use them in the form (due to the term $\nabla\vec{\xi}$, see e.g. Ref. [31])

$$\partial/\partial t\,(\partial L/\partial\dot{q}) - \delta L/\delta q = 0 \tag{32}$$

where the functional derivative

$$\frac{\delta L}{\delta q} = \frac{\partial L}{\partial q} - \frac{\partial}{\partial x}\frac{\partial L}{\partial(\partial q/\partial x)} - \frac{\partial}{\partial y}\frac{\partial L}{\partial(\partial q/\partial y)} - \frac{\partial}{\partial z}\frac{\partial L}{\partial(\partial q/\partial z)}. \tag{33}$$

The equations for m and $\vec{\xi}$ obtained from Eqs. (32) and (33) are the following

$$\partial^2 m/\partial t^2 - (m_0/T)\nabla\dot{\vec{\xi}} = 0\,, \tag{34}$$

$$\partial^2\vec{\xi}/\partial t^2 - (\Lambda^2/m_0 T)\ \nabla\dot{m} = 0\,. \tag{35}$$

To solve equations (34) and (35), we have to set initial conditions to variables $m(\vec{r},\,t)$ and $\vec{\xi}(\vec{r},\,t)$. It is obvious that the initial conditions are

$$m(\vec{r},\,0) = m(\vec{r}), \qquad \partial\, m/\partial\, t\big|_{t=0} = 0\,; \tag{36}$$

$$\vec{\xi}(\vec{r},\,0) = \vec{\xi}(\vec{r}), \qquad \partial\vec{\xi}/\partial\, t\big|_{t=0} = 0\,. \tag{37}$$

The boundary conditions are

$$m\,(l_{\text{Planck}},\,t) = m(t), \quad m\,(\Lambda,\,t) = 0, \quad \partial\, m/\partial\,\vec{r}\,\big|_{r=l_{\text{Planck}}} = 0; \tag{38}$$

$$\vec{\xi}\,(l_{\text{Planck}},\,t) = 0, \quad \vec{\xi}\,(\Lambda,\,t) = \vec{\xi}(t), \quad \frac{\partial\vec{\xi}}{\partial\vec{r}}\bigg|_{r=\text{Planck}} = F(t), \quad \frac{\partial\vec{\xi}}{\partial\vec{r}}\bigg|_{r=\Lambda} = 0\,. \tag{39}$$

The conditions (36) to (39) show that the mass is initially has located in the centre of coordinates of the system studied, i.e. in the particle, such that $m\,(\vec{r},\,t)\,|_{l_{\text{Planck}},\ 0} \;=\; m_0/\sqrt{1-\upsilon^2/c^2}$. Besides, these conditions mean that the behavior of the mass of the {particle + inerton cloud}-system and that of the rugosity of the tessel-lattice are opposite in phase.

The conditions (36) to (39) allow one to transform equations (34) and (35) to the form (Δ is the Laplace operator)

$$\partial^2 m / \partial t^2 - c^2 \Delta m = 0, \tag{40}$$

$$\partial^2 \vec{\xi} / \partial t^2 - (\Lambda^2 / m_0 T) \nabla \dot{m} = 0. \tag{41}$$

Since the system studied features the radial symmetry, variables m and $\vec{\xi}$ are functions of only the distance r from the particle and the proper time t of the {particle + inerton cloud}-system. In this case we preserve only radial components in the both variables, which enables to rewrite equations (40) and (41) in the spherical coordinates as follows

$$\partial^2 m / \partial t^2 - (c^2 / r) \partial^2 (r m) / \partial r^2 = 0, \tag{42}$$

$$\partial^2 \xi / \partial t^2 - (\Lambda^2 / m_0 T)(\partial / \partial r) \partial m / \partial t = 0 \tag{43}$$

where the Laplace operator Δ is presented in the spherical coordinates as $\Delta m = \frac{1}{r} \frac{\partial^2}{\partial r^2}(rm)$.

Thus the availability of the radial symmetry and the conditions (36) to (39) allow the solutions to equations (42) and (43) in the form of standing spherical waves, which exhibit the dependence $1/r$,

$$m(r,\ t) = (C_1 m_0 / r) \cos(\pi r / 2\Lambda) \left|\cos(\pi t / 2T)\right|, \tag{44}$$

$$\xi(r,\ t) = (C_2 \xi_0 / r) \sin(\pi r / 2\Lambda)\ (-1)^{[t/T]} \sin(\pi t / 2T) \tag{45}$$

where we omit the radical $\sqrt{1 - \upsilon^2 / c^2}$ at the mass m_0 and introduce the notation $[t/2T]$ meaning an integral part of the integer $t/2T$. The dimensionality of integration constants $C_{1,2}$ corresponds to length and one can put $C_1 = l_{\text{Planck}} \approx 10^{-35}$ m and $C_2 = \Lambda$.

The solution (44) shows that the amplitude of mass of the inerton cloud

$$m_0 / r \cos(\pi r / 2\Lambda), \tag{46}$$

represents the particle mass distribution in the range from $r = l_{\text{Planck}}$ to $r = \Lambda$ where $\Lambda = \lambda\, c / \upsilon$ is the amplitude of the particle's inerton cloud.

The solution (45) depicts a similar pattern for the behavior of the rugosity, or tension, of the tessel-lattice in ambient space.

In the region of space $l_{\text{Planck}} << \; r \; << \Lambda$, the time-averaged distribution of the {particle + inerton cloud}-mass becomes

$$m\,(r) \;\cong\; l_{\text{Planck}}\; m_0 / r\,. \tag{47}$$

In this region of space, the tension of space, as follows from expression (45), $\xi(r) \cong 0$.

Thus in mathematics terms, one can say that a local fractal deformation of the space tessel-lattice when moves induces the rugosity in the tessel-lattice. When the local deformation is distributed in space, it forms a deformation potential $\propto 1/r$ that spreads up to a distance $r = \Lambda$ from the core-cell, i.e. particle. *In the range covered by the deformation potential, cells of the tessel-lattice are found in the contraction state and it is this state of space, which is responsible for the phenomenon of the gravitational attraction.*

In physics terms, the total mass (30) of the particle periodically decays (in the section equals the de Broglie wavelength λ) transferring to the inertons cloud that induces the mass field, i.e. deformation potential, $\propto 1/r$ (47), in the range up to a distance $r < \Lambda,$ which is identical to the gravitational potential of the particle.

Does the distribution of mass field (47) of an isolated particle lead to Newton's potential of a macroscopic object? Yes, it does. A macroscopic object is a many-particle system in which particles (atoms is a solid or protons in a star) are tied by means of their inerton clouds in a uniform system. Such unification gives rise to coherent collective vibrations of atoms/protons (acoustic vibrations). As has been argued early [32], those are amplitudes A of vibrating atoms that in a condensed matter play the role of the de Broglie wavelength λ of free particles. Therefore in a condensed matter relation (21) is replaced by

$$\Lambda = A\,c / \upsilon \tag{48}$$

where the velocity υ should be the same for every atom, because this is the sound velocity (the group velocity of vibrating atoms in acoustic modes). The amplitude A depends on the number of atoms involved in a vibrating mode, i.e. $A_s = 2A_1 \cdot s / \aleph$ where $s = 2, 3, ..., (\aleph - 1)$ is the number of atoms in the *s*th mode and $\aleph$ is the total number of object's atoms. In such a way A_s should be regarded as the wavelength of *s*th mode/harmonic of the object studied. The longest mode corresponds to collective vibrations of all atoms of the object, which means that all $\aleph$ atoms generate the total collective inerton cloud with the wavelength $A_\aleph$ in the ambient space. Thus expression (47) holds also for the rest mass of a macroscopic object.

Multiplying both parts of expression (47) by a factor $-G / l_{\text{Planck}}$ where G is the Newton constant of gravitation, we obtain the so-called potential gravitational energy, or Newton's gravitational potential, of the object in question

$$U(r) = -G\ m_0 / r \tag{49}$$

where the Newton constant simple plays the role of a dimensional coefficient. As provided by relation (48), the potential (49) spreads from the object to a huge distance $r < \Lambda_\aleph c / \upsilon \sim (V_{\text{object}} / v_{\text{particle}})\, c / \upsilon$, where V_{object} and v_{particle} are typical volumes of the object and the object's effective particle, respectively.

7. Experimental verification

To check the model of gravitation described above one should test excitations of the space tessel-lattice considering in the given model as a primary substrate. Let us dwell on major experimental results available at present.

7.1. Resonator of inertons. The impact of inerton waves on the behavior of atoms in metals was studied theoretically and then observed experimentally as changes in the fine morphological structure of specimens by the high-resolution electron-scanning microscope in paper [33]. In the experiment we used a resonator of inerton waves of the Earth, which represented a small projective model of the terrestrial globe. In essence the paper is the first reliable demonstration of the existence of the so-called aether wind generated by the Earth at its motion through the world aether, or space that is most appropriate for our consideration.

7.2. Anomalous photo-electric effect. In paper [34] electrons moving in atoms were treated as entities surrounded by their inerton clouds. The investigation of the interaction between such entities and a photon flux was curried out in detail. The major peculiarity of the theory proposed was the effective cross-section of electrons significantly enlarged due to their inerton clouds spread around the electrons.

It was shown that a number of different experiments aimed at the study of laser-induced gas ionization were in agreement with the theoretical results prescribed by the inerton theory, namely: (a) the concentration of ionized atoms was directly proportional to the peak laser pulse intensity and the time to the second power; (b) the prediction of temporal dependence of the threshold intensity was in accord with the experimental data; (c) the experimental results on the number of ions created by the laser pulse as a function of the pulse intensity checked well with the description in terms of the anomalous photo-electric effect constructed; (d) the breakdown intensity threshold measured as a function of pressure or gas density fitted well with the theory; (e) an anomalous value of electrons released from atoms of gas at high energies of the laser beam correlated with the inerton theory.

Electron emission from a laser-irradiated metal was also correctly described by the inerton theory, namely, the theory was in consistent with (a) the experimental results indicated that the photo-electric current was a linear with light intensity; (b) the data showing that the maximum energy of the emitted electron is a function of light intensity.

7.3. Oscillations of hydrogen atoms. The inerton concept was justified [32] at the employing for the experiment on the study of fine dynamics of hydrogen atoms in the $\mathrm{KIO_3 \cdot HIO_3}$ crystal whose FT IR spectra in the 400 to 4000 cm^{-1} spectral range showed unexplainable sub maximums. Features observed in the spectra were unambiguously interpreted [32] as caused by the hydrogen atoms' matter waves that induced the mean inerton field contributing to the paired potential of hydrogen-hydrogen interaction. This admitted solution in the cluster form. The number of hydrogen atoms entered a cluster was assessed and its spectrum was calculated. It was inferred [32] that those were sub maximums in the total spectrum of the crystal, which emerged due to the collective oscillations of hydrogen atoms interacting in clusters through the inerton field.

7.4. A deviation from the inverse square law for gravity. This theme attracts a wide attention of those researchers who try to test gravity in the microscopic range. It is assumed [35] that a new kind of interaction coexists with the conventional gravity, which has to modify Newton's gravitational law. The non-Newtonian potential is chosen in the form of the Yukawa potential in such a way that the modified Newtonian potential for masses m_i and m_j is written as follows

$$U(r) = -(G\, m_i\, m_j / r)\ (1 + \alpha\, e^{-\,r/Y}) \tag{50}$$

where Y is the Yukawa distance over which the corresponding force acts and α is a strength factor in comparison with Newtonian gravity.

In the recent experiment [36] ultra-cold neutrons falling in the Earth gravitational field are reflected and trapped in a cavity above a horizontal mirror. It was revealed that the population of the ground state and the lowest neutron states followed the quantum mechanical prediction (the higher, unwanted neutron states were removing by an efficient absorber). This very interesting precise experiment showed the availability of quantum states for Newtonian gravity on the micrometer scale, which allowed the authors to place limits for an additional gravity-like force in the range from 1 to 10 μm with an explicit quantum tail up to 70 μm.

So, what is the origin of the correction introduced in expression (50) to the classical Newton's law? Let us try to answer the question drawing inertons.

Let in expression (50) m_i stands for the Earth mass $m_{\rm Earth}$ and m_j stands for the neutron mass $m_{\rm n}$. In the zero approximation, the Earth gravitational field attracts a test point particle, i.e. neutron, in accordance with the classical Newton's gravitational law,

$$U(r) = -G\ m_{\rm Earth}\, m_{\rm n} / r \tag{51}$$

However, in the next approximation we should take into account that the neutron is not a point-like particle. In fact, in the experiment described in Ref. [36] the neutron velocity was around several meters per second such that the de Broglie wavelength λ varied from 40 nm to 100 nm. In conformity with relationship (21) the neutron's inerton cloud spreads in directions transversal to the neutron path up to $\Lambda = \lambda c / \upsilon$ ~ 1 m and this is the real gravitational radius of the neuron. This means that the neutron moving along its path falls via its inerton cloud within the exchange gravitational interaction with the environment. Let M be the effective mass of the nearest environmental matter that surrounds the moving neutron and μ be the mass transferable by the neutron's inerton cloud. Then the following "kinetic" equations can be proposed

$$d\mu / dr = a\mu - bM, \tag{52}$$

$$dM / dr = kM - a\mu . \tag{53}$$

Here in Eq. (52) the terms $a\mu$ and bM respective describe the decay of the inerton cloud's mass of a moving neutron and the restoration of the inerton cloud's mass due to the mass input on the side of the environment; in Eq. (53) the terms kM and $a\mu$ describe the outflow of the mass from the environment and the input mass to the environment from the neutron's inerton cloud, respectively. The variable r in Eqs. (52) and (53) characterises the distance of the moving neutron to the detector (the Earth surface). Equations (52) and (53) result in the following equation for the inerton cloud's mass

$$d^2\mu / dr^2 - (a+k) d\mu / dr - a \cdot (k-b)\mu = 0, \tag{54}$$

Eq. (54) has the solution

$$\mu(r) = \mu_{\text{eff}} \exp\left\{ -\left((a+k)/2 + \sqrt{(a+k)^2/4 + a(k-b)} \right) r \right\} \tag{55}$$

where, as clear from Eqs. (52) and (53), $k - b > 0$. Expr. (55) can be rewritten in the contracted form,

$$\mu(r) = \mu_{\text{eff}} \exp(-Yr) \tag{56}$$

and then the mass m_{n} of the neutron in expression (51) should be added by the correction (56),

$$m_{\text{n}} \rightarrow m_{\text{n}} + \mu_{\text{eff}} \exp(-Yr), \tag{57}$$

which immediately results in the modified Newton's law (50). In the experiment [35] the distance Y was estimated as equal to several micrometers. Note that the parameters entered expression (55) allows the evaluation in principle.

Consequently, the origin of the Yukawa potential in expression (50) arises from the exchange of mass of the inerton cloud of a falling particle with a substance surrounding the particle.

7.5. Circumstantial confirmations. In paper [37] the authors claimed that they observed wave ψ-functions of electrons on metal surfaces. This automatically means that in fact the wave ψ-function is not an abstract mathematical construction, but a function that characterizes the real perturbation of space surrounding the electron in the metal, i.e. the inerton cloud of the electron.

Podkletnov [38,39] has made a demonstration of the loss of a part of the mass of electrons in the superconductor state at special conditions, which can serve as a good example supporting our theory of quantum gravity that emerges due to the inerton interaction between quantum entities.

Changes in the structure of metal samples irradiated by a field of undetermined nature (the so-called “torsion fields”) are displayed in book [40] (the results by V. Maiboroda, A. Akimov et al.). The torsion radiation was introduced pure theoretically by Shipov [40] as a primary field that allegedly was dominating over a vague physical vacuum long before its creation. Nevertheless, the named authors experimenting with metals have used as a source of this new field, substantive generators, which allows us to suggest that they have dealt with the source of the inerton waves.

Just as the author, Baurov [41] sees all particles as being conceived from a unique corpuscle; moreover, some direct experimental evidences of the interaction of matter with a sub quantum medium has indeed been demonstrated by him.

Experimentalists who investigate low energy nuclear reactions claim that they fix a new “strange” radiation at the transmutation of nuclei, Urutskoev et al. [42]; possibly the “strange” radiation is the inerton field that is radiated from the appropriate inerton clouds of nucleons when the latter are rearranging in nuclei. Gauging the radiation of an unknown field from a rotating ferrite disc was performed by Benford [43]; the work provides a demonstration of the transmutation of chemical entities under this unknown radiation.

7.6. Further experimental protocols. Former experiments by several researchers [44,45] have confronted the recoding of informational signals from stars other than electromagnetic ones, which is known as Kozyrev’s effect. We have proposed [46] a series of protocols for testing the inerton radiation from stars and planets; the measurements will be conducted by means of special pyroelectric sensors constructed for this purpose, which will be embedded in the focal volume of the telescope. The first encouraging measurement of the inerton radiation was in fact obtained along the west-east line at 20 Hz, which was associated with proper rotation of the globe; besides, inerton signals at frequencies 18 to 22 kHz, which came from the northern sky in a universal time interval from 3 p.m. to 5 p.m., were recorded from September 2004 to February 2005.

We [47] have just examined the phenomenon of radiation of physical fields generated by the so-called Teslar watch on organic matter. The Teslar watch contains a special chip that generates an unknown field called *the scalar Teslar wave*; the Teslar watch's radiation very positively influences the human organism preserving it from the environmental low frequency electromagnetic pollution, supports good health, facilitates a fast relaxation of the nervous system, and so on. Our study in fact has justified that the Teslar watch generates nor the electromagnetic, neither ultrasound radiation. This is the inerton radiation generated by a special electric circuit, or the Teslar chip, embedded in the watch (two superimposed electromagnetic waves whose amplitudes are shifted to 180^{o} are cancelled, but an inerton flow that continues to transfer the energy remains. This is quite possible, as it follows from the study of the phenomenon of the electric charge and principles of its motion in the tessel-lattice [48]). Five different experiments were carried out [47].

At last the inerton field is the fundamental one in nuclear physics; it is this field that bounds nucleuses in nuclei accounting for the reasons for the nuclear forces playing a role of a control field in nuclear physics. The reconsideration of basic principles of nuclear physics has allowed for a radically new approach to completely clean nuclear energy [49].

8. Concluding remarks

In the present paper based on the rigorous mathematical approach initiated by Michel Bounias we have disclosed the founding principles of the constitution of the physical space and shed light on the foundation of space-time. We have introduced the determination of physical space as the fractal tessel-lattice that is characterised by a primary element (a topological ball, or a cell, or a superparticle) and defined the notion of mass and hence a particle as a local deformation of space. The interaction of a moving particle-like deformation with the surrounding tessellattice involves a fractal decomposition process that supports the existence and properties of previously introduced inerton clouds as associated to particles.

The further study has allowed one to account for the inertial and the gravitational masses of a particle, which in fact in line with de Haas' [22] research aimed at the unification of Mie's theory of gravitation and de Broglie's wave mechanics. It has been shown that the so-called relativistic mass (30) is responsible for the induction of the gravitational potential of a moving canonical particle: The total particle mass (30) is allocated in the particle's inerton cloud transferring an additional reduction to surrounding cells of the tessellation space. The particle's inertons spread to the range $\Lambda = \lambda c / \upsilon$ from the particle where υ and λ respective are the particle's velocity and de Broglie wavelength. Therefore, a canonical particle with a mass m being moving generates the mass field, or the gravitational mass, that obeys the law (47); this consideration remains valid in the case of a classical object.

The particle's inertons migrate from cell to cell by relay-race mechanisms, i.e. hops from cell to cell, and spread to the distance Λ as a typical advancing spherical wave. Since any spherical wave is characterized by the inverse law $1/r$, this immediately results in the distribution of the inerton field proportional to $1/r$ around the particle, or a classical object. This is the inner reason for Newton's gravitational law. Thus the phenomenon of gravitational attraction becomes complete clear: Inertons carry fragments of the inertial deformation, i.e. mass, of the object to the surrounding space forming a deformation potential, i.e. the mass field, in which each cell of the tessellattice undergoes an additional contraction in accord with the rule (47).

The quantization of the gravitation is caused by the periodical process of emission and absorption of inerton clouds by a moving particle. This process intrudes itself upon the space in which the motion occurs. Any cyclic motion allows the description in terms of the Hamilton-Jacobi formalism. In our case the minimum of increment of the action becomes equals to Planck's constant *h*, which means that it is the space that exerts control over the behaviour of the particle [16,17]. Hence submicroscopic mechanics is in fact quantum and moreover is readily transformed to conventional quantum mechanics developed in an abstract (phase, Hilbert, and etc.) space.

The submicroscopic concept forbids the existence of so-called gravitational waves of general relativity [33]. Loinger [50] has proved the same, though he has started from the conventional Gilbert-Einstein equations and shown that Einstein's gravitational waves are not a realistic solution.

The submicroscopic concept stated above has successfully been verified experimentally, but rather in microscopic and intermediate ranges. The question arises whether the theory can be adjusted with general relativity that works in a macroscopic range. Probably the submicroscopic concept can be able to account for those crucial experiments that have been formally predicted by general relativity, namely: The motion of the perihelion of a planet, the deviation of light by a star, and the red shift of light under the gravitation. These studies will be conducted in the nearest future.

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