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Submicroscopic Deterministic Quantum Mechanics

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Abstract. So-called hidden variables introduced in quantum mechanics by Louis de Broglie and David Bohm have been revived in the recent works by the author. The start viewpoint was the following: All the phenomena, which we observe in the quantum world, should reflect structural properties of the real space. Thus the scale 10^{-28} cm at which fundamental interactions intersect has been treated as the size of a building block of the space. The mechanics of a moving particle that has been constructed is deterministic by its nature and shows that the particle interacts with cells of the space creating elementary excitations called "inertons" in a cellular substrate. The existence of inertons has been verified experimentally.

Key words: space structure, particle, inertons (elementary excitations), quantum mechanics

1 Conceptual difficulties of quantum theory

The main original physical parameters of quantum theory are Planck's constant h and de Broglie's wavelength λ . These two enter into the two major quantum mechanical relationships for a particle proposed by Louis de Broglie (see, e.g. de Broglie, 1986)

$$E = h\nu; \quad \lambda = h/p. \quad (1)$$

De Broglie believed that E and p were the energy and the momentum of the particle, ν was the peculiar particle's frequency that coincided with the frequency of a wave that specified by the wavelength λ and traveled together with the particle. Later when Schrödinger's equation appeared and Heisenberg proposed the uncertainty relations,

the interpretation of the said characteristics changed. Namely, the notion of the particle was transformed to a "particle-wave" and hence and became characteristics of the particle-wave. Born interpreted the square of the absolute magnitude of the wave ψ -function of the Schrödinger equation as the probability of particle location in a place described by the radius vector \mathbf{r} . Thus Born finally rejected any physical interpretation from a set of parameters that described a quantum system. Since the end of the 1920s, only one parameter has been perceived as pure physical - the particle-wave wavelength λ called the de Broglie wavelength. In experimental physics, those waves received also another name – the matter waves. Such a name directly says that corpuscles (but not dim "particle-waves") are able to manifest a wave behavior.

Since 1952 de Broglie followed two papers by Bohm (1952) (see also Bohm, 1996) turned back to his initial ideas on the foundations of the wave mechanics of particles. De Broglie (see, e.g. de Broglie, 1960, 1987) believed that a submicroscopic medium interfered with the motion of a particle and the appropriated wave guided the particle. He firmly believed the causal interpretation of quantum mechanics and warned that the resolution of the issue should not be based on the wave ψ -function formalism, as the ψ -function was determined only in the phase space but not in a real one. His own attempts were aimed at seeking for the form of the so-called double solution.

In the case of the Dirac formalism things get worse. The formalism introduced new additional notions such as spinors and Dirac's four-row matrices, which allowed the calculation of the energy states of the quantum system studied and changes in the states due to the influence of outside factors. However, the formalism did not propose any idea on the reasons of the wave behavior of matter and a nature of the particle spin.

So far, modern studies devoted to the foundations of quantum mechanics have tried to reach the deepest understanding of quantum theory reasoned that just the ψ -function formalism is original and it is often exploited even on the scale of Planck length $\sqrt{G\hbar/c^3} \sim 10^{-33}$ cm. This is especially true for quantum field theory including quantum gravity (see, e.g. Wallace, 2000; Sahni and Wang, 2000). Besides, there are views that a gravitationally induced modification to the de Broglie's wave-particle duality is needed when gravitational effects are incorporated into the quantum measurement process (Ahluwalia, 1994, 2000; Kempf et al., 1995). Other approaches try to introduce a phenomenological description based on the metric tensor g_{ij} in typical quantum problems ('t Hooft, 1998). Classical Einstein gravity is also exploited in condensed matter: some parameters such as mass, spin, velocity, etc. are combined to provide an effective "metric" that then is entered into the quantum mechanical equations (e.g. Danilov et al. (1996) and Leonhard and Piwnicki (1999)).

Thus, the trend has been forward to the entire intricacy: the formalism of ψ -function penetrates to the Planck length interior and the Einstein metric formalism advances to the same scale as well. Nobody wishes to accept the fact that on the size comparable with the de Broglie wavelength λ of an object methods of general relativity fail. No one wants to go deeply into de Broglie's remark that the ψ -function is only a reflection of some hidden variables of a particle moving in the real physical space. The ψ -function is not the mother of particle nature and therefore it cannot serve as a variable of the expansion of a particle's characteristic in terms of ψ at the

size less than the particle's de Broglie wavelength λ .

2 New understanding

Among new approaches describing gravity in the microworld, we can notice the mathematical knot theory (see, e.g. Pullin, 1993), which has been developed (Wallace, 2000) attempting to find rules to establish when one knot can be transformed into another without untying it. In the theory, the question is reduced to a certain knot invariant problem, which does not change with knot deformations; knot invariants being deformed constantly by gauge transformations should stay unchangeable. The approach is similar in many aspects to concepts elaborated in elementary particle physics.

Of special note is the approach proposed by Bounias (1990, 2000) and Bounias and Bonaly (1994, 1996, 1997). Basing on topology and set theory, they have demonstrated that the necessity of the existence of the empty set leads to the topological spaces resulting in a "physical universe". Namely, they have investigated links between physical existence, observability, and information. The introduction of the empty hyperset has allowed a preliminary construction of a formal structure that correlates with the degenerate cell of space supporting conditions for the existence of a universe. Besides, among other results we can point to their very promising hypothesis on a non-metric topological distance as the symmetric difference between sets: this could be a good alternative to the conventional metric distance which so far is still treated as the major characteristic in all concepts employed in gravitational physics, cosmology, and partly in quantum physics.

In my own line of research I started from the fact that on the scale $\sim 10^{-28}$ cm constants of electromagnetic, weak and strong interactions as functions of distance between interacting particles intersect (see, e.g., Okun, 1988). On the other hand, in the high energy physics theorists deal with an abstract "superparticle" whose different states are electron, muon, quark, etc. (see, e.g. Amaldi, 2000). A simple logical deduction suggests by itself: the physical space at the said range has a peculiarity that could be associated with presence of structural blocks which one can call just superparticles (or elementary cells, or balls). Then one may expect that a theory of the physical space densely packed with those superparticles will be able to overcome many difficulties which are insuperable in formal theories of both quantum gravity and high energy physics. Thus a submicroscopic theory being based on the structure of fine-grained space will be able to widely expand our knowledge about the origin of matter, the foundations of quantum mechanics and the foundations of quantum gravity.

The first step of the theory, Krasnoholovets and Ivanovsky (1993), Krasnoholovets (1997, 2000a, 2000b), focused on the appearance of a particle from a superparticle, which initially was found in the degenerate state. The particle has been defined as a local curvature, or a local deformation of a superparticle and hence the appearance of the deformation in a superparticle means the induction of mass in it, $m \propto CV_{\text{sup}}/V_{\text{part}}$ (C is the dimensional constant, V_{sup} is the initial volume of a degenerate superparticle

and V_{part} is the volume of the deformed superparticle, i.e. the volume of the created particle). So the real space was regarded rather as a substrate, or "quantum aether", and the notion of a particle in it was adequately determined.

In condensed matter, we meet the effect of the deformation of the crystal lattice in the surrounding of a foreign particle and the solvation effect in liquids. Therefore, the second step of the theory was the proposition that around a particle a deformation coat was induced. This coat should play the role of a screen shielding the particle from the degenerate space substrate. Within the coat, the space substrate should be considered as a crystal and superparticles here feature mass. Thus the coat may be treated as a peculiar crystallite. The size of the crystallite was associated with the Compton wavelength of the particle, $\lambda_{\text{Com}} = h/mc$.

The next step needed a correct physical model of the motion of the particle. From the solid state physics we know that the motion of particles is accompanied with the motion of elementary excitations of some sort, namely, the particle when it moves in a solid emits and absorbs quasi-particles such as excitons, phonons, etc. By analogy, the motion of the physical "point" (particle cell) in the entirely packed space must be accompanied by the interaction with surrounding coming "points" of the space, i.e. superparticle cells. Hence the particle is scattered by structural blocks of the space that in turn should lead to the induction of elementary excitations in superparticles, which contact the moving particle. The corresponding excitations were called "inertons" as the notion "inertia" means the resistance to the motion (thus particle's inertons reflect resistance on the side of the space in respect to the moving particle). Each inerton carries a bit of the particle deformation, that is, an inerton is characterized by the mass as well. An inerton migrates from superparticle to superparticle by relay mechanism. The deformation coat, or crystallite (by analogy with crystal physics), is pulled by the particle: superparticles, which form the crystallite, do not move from their positions in the space substrate, however, the massive state of crystallite's superparticles is passed on from superparticles to superparticles along the whole particle path.

3 Submicroscopic mechanics

The Lagrangian that is able to satisfy the described motion of a particle and the ensemble of its inertons can be written as (Krasnoholovets and Ivanovsky, 1993)

$$L = \frac{1}{2}g_{ij}\frac{dX^i}{dt}\frac{dX^j}{dt} + \frac{1}{2}\sum_s g_{ij}^{(s)}\frac{dx_{(s)}^i}{dt_{(s)}}\frac{dx_{(s)}^j}{dt_{(s)}} - \sum_s \delta_{t-\Delta t_{(s)}, t_{(s)}} \frac{\pi}{T_{(s)}} \left[X^i \sqrt{g_{iq}\theta\tilde{g}_{(s)qj}} \frac{dx_{(s)}^j}{dt_{(s)}} + (\mathbf{v}_0)^i \sqrt{g_{iq}\theta\tilde{g}_{(s)qj}} x_{(s)}^j \right] \quad (2)$$

where the first term characterizes the kinetics energy of the particle, the second term characterizes the kinetics energy of the ensemble of N inertons, emitted from the particle and the third term specifies the contact interaction between the particle and its inertons. X^i is the i th component of the position of the particle; g_{ij} is metric

tensor components generated by the particle; $(\mathbf{v}_0)^i$ is the i th component of the initial particle's velocity vector \mathbf{v}_0 . Index s corresponds to the number of respective inertons; $x_{(s)}^j$ is the component of the position of the s th inerton; $g_{(s)ij}$ is the metric tensor components of the position of the s th inerton. $1/T_{(s)}$ is the frequency of collisions of the particle with the s th inerton. Kronecker's symbol $\delta_{t-\Delta t_{(s)}, t_{(s)}}$ provides the agreement of proper times of the particle t and the s th inerton $t_{(s)}$ at the instant of their collision ($\Delta t_{(s)}$ is the time interval after expire of which, measuring from the initial moment $t = 0$, the moving particle emits the s th inerton). The interaction operator $\sqrt{g_{iq} \theta \tilde{g}_{(s)qj}}$ possesses special properties: $\theta = 0$ during a short time interval δt when the particle and the s th inerton is in direct contact and $\theta = 1$ when the particle and the s th inerton fly apart along their own paths. Note that in the model presented the metric tensor characterizes changing in sizes of the particle and superparticles.

In the so-called relativistic case when the initial velocity \mathbf{v}_0 of the particle is close to the speed of light c , the relativistic mechanics prescribes the Lagrangian

$$L_{\text{rel}} = -M_0 c^2 \sqrt{1 - \mathbf{v}_0^2/c^2}. \quad (3)$$

On examination of the relativistic particle, we shall introduce into the Lagrangian (3) terms, which describe inertons and their interaction with the particle. For this purpose, the following transformation in (3) should be made (Krasnoholovets, 1997)

$$L_{\text{rel}} = -g c^2 \left\{ 1 - \frac{1}{g c^2} \left[g_{ij} \frac{dX^i}{dt} \frac{dX^j}{dt} + \sum_s g_{ij}^{(s)} \frac{dx_{(s)}^i}{dt_{(s)}} \frac{dx_{(s)}^j}{dt_{(s)}} - \sum_s \delta_{t-\Delta t_{(s)}, t_{(s)}} \frac{\pi}{T_{(s)}} \left(X^i \sqrt{g_{iq} \theta \tilde{g}_{(s)qj}} \frac{dx_{(s)}^j}{dt_{(s)}} + (\mathbf{v}_0)^i \sqrt{g_{iq} \theta \tilde{g}_{(s)qj}} x_{(s)}^j \right) \right] \right\} \quad (4)$$

where $g = g_{ij} \delta^{ij}$.

The Euler-Lagrange equations

$$\frac{d}{dt_{(s)}} \frac{\partial \mathcal{L}}{\partial (dQ/dt_{(s)})} - \frac{\partial \mathcal{L}}{\partial Q} = 0 \quad (5)$$

written for the particle ($Q = X^i$) and the s th inerton ($Q = x_{(s)}^i$) coincide for the Lagrangians $\mathcal{L} = L$ (3) and $\mathcal{L} = L_{\text{rel}}$ (4). This is true only (Dubrovin et al. (1986)) in the case when the time t entered into the Lagrangians (3) and (4) is considered as the natural parameter, i.e. $t = l/v_0$ where l is the length of the particle path.

For the variables $X_{(s)}^k = X^k(t_{(s)})$ and $x_{(s)}^k = x^k(t_{(s)})$ one obtains from eq. (5) the equations of extremals (written as functions of the proper time $t_{(s)}$ of the emitted s th inerton):

$$\frac{d^2 X_{(s)}^k}{dt_{(s)}^2} + \Gamma_{ij}^k \frac{dX_{(s)}^i}{dt_{(s)}} \frac{dX_{(s)}^j}{dt_{(s)}} + \frac{\pi}{T_{(s)}} g^{ki} \sqrt{g_{iq} \theta \tilde{g}_{(s)qj}} \frac{dx_{(s)}^j}{dt_{(s)}} = 0; \quad (6)$$

$$\frac{d^2 x_{(s)}^k}{dt_{(s)}^2} + \tilde{\Gamma}_{(s)ij}^k \frac{dx_{(s)}^i}{dt_{(s)}} \frac{dx_{(s)}^j}{dt_{(s)}} - \frac{\pi}{T_{(s)}} \left[g_{(s)}^{ki} \sqrt{g_{iq} \theta_{(s)qj}} \left(\frac{dX_{(s)}^j}{dt_{(s)}} - (\mathbf{v}_0)^j \right) \right] = 0; \quad (7)$$

here, Γ_{ij}^k and $\tilde{\Gamma}_{(s)ij}^k$ are symmetrical connections (see, e.g. Dubrovin et al., 1996) for the particle and for the s th inerton, respectively; indices i, j, k and q take values 1, 2, 3. When the particle and the s th inerton adhere, the operator $\theta = 0$ and therefore the termwise difference between eqs. (6) and (7) becomes

$$\left(\frac{d^2 X_{(s)}^k}{dt_{(s)}^2} - \frac{d^2 x_{(s)}^k}{dt_{(s)}^2}\right) + \left(\Gamma_{ij}^k \frac{dX_{(s)}^i}{dt_{(s)}} \frac{dX_{(s)}^j}{dt_{(s)}} - \tilde{\Gamma}_{(s)ij}^k \frac{dx_{(s)}^i}{dt_{(s)}} \frac{dx_{(s)}^j}{dt_{(s)}}\right) = 0. \quad (8)$$

Eq. (8) specifies the merging the particle and the s th inerton into a common system. This means the acceleration that the particle experiences, coincides with that of the s th inerton. Then the difference in the first set of parentheses in eq. (8) is equal to zero and instead of eq. (8) we get

$$\Gamma_{ij}^k \frac{dX_{(s)}^i}{dt_{(s)}} \frac{dX_{(s)}^j}{dt_{(s)}} = \tilde{\Gamma}_{(s)ij}^k \frac{dx_{(s)}^i}{dt_{(s)}} \frac{dx_{(s)}^j}{dt_{(s)}}. \quad (9)$$

Coefficients Γ_{ij}^k and $\tilde{\Gamma}_{(s)ij}^k$ are generated by the particle mass M and the s th inerton mass $m_{(s)}$, respectively, and that is why $\Gamma_{ij}^k/\tilde{\Gamma}_{(s)ij}^k = M/m_{(s)}$. This signifies that relationship (9) can be rewritten explicitly

$$Mv_{0s}^2 = m_{(s)}c^2 \quad (10)$$

for diagonal metric components of the particle and inerton velocities, ($v_{0(s)}$ is the velocity of the particle after its scattering by the s th inerton with initial velocity c).

When the particle and the s th inerton bounce apart, we must solve the total equations of motion, (6) and (7), i.e. all terms in the equations should be held. However, if we allow the metric tensors to be constant, the equations of motion may be simplified to the form that does not include the second nonlinear term in both eqs. (6) and (7). The structure and properties of the metric tensors can be chosen as follows

$$\begin{aligned} g_{ij} &= \delta_{ij}M; & g^{ij} &= \delta^{ij}/M; & g_{ki}g^{iq} &= \delta_k^q; \\ \tilde{g}_{(s)ij} &= \delta_{ij}m_{(s)}; & \tilde{g}_{(s)}^{ij} &= \delta^{ij}/m_{(s)}; & \tilde{g}_{(s)ki}\tilde{g}_{(s)}^{iq} &= \delta_k^q. \end{aligned} \quad (11)$$

Thus having given g_{ij} and $g_{(s)ij}$ are equal to constant, the second term in both eqs. (7) and (8) is made to be reduced to zero. Relationships (11) and (10) allow transforming of the interaction operator in eqs. (7) and (8) to forms

$$g^{ki} \sqrt{g_{iq} \theta \tilde{g}_{(s)qj}} \longrightarrow \sqrt{\frac{m_{(s)}}{M}} = \frac{v_{0(s)}^k}{c}; \quad (12)$$

$$g^{ki} \sqrt{g_{iq} \theta \tilde{g}_{(s)qj}} \longrightarrow \sqrt{\frac{M}{m_{(s)}}} = \frac{c}{v_{0(s)}^k} \quad (13)$$

where $v_{0(s)}^k$ is the k th component of the vector $\mathbf{v}_{0(s)}$. Thus expressions (12) and (13) permit the transformation of eqs. (7) and (8) (in which second terms are dropped) to the form

$$\frac{d^2 X_{(s)}^k}{dt_{(s)}^2} + \frac{\pi v_{0(s)}^k}{c T_{(s)}} \frac{dx_{(s)}^j}{dt_{(s)}} = 0; \quad (14)$$

$$\frac{d^2 x_{(s)}^k}{dt_{(s)}^2} - \frac{\pi c}{v_{0(s)}^k T_{(s)}} \left[\frac{dX_{(s)}^k}{dt_{(s)}} - (v_{0(s)})^k \right] = 0. \quad (15)$$

Initial conditions are

$$\frac{dX_{(s)}(t_{(s)} + \Delta t_{(s)})|_{t_{(s)}=0}}{dt_{(s)}} = \frac{dX_{(s)}(\Delta t_{(s)})}{dt_{(s)}} = v_{(s)0};$$

$$x_{(s)}|_{t_{(s)}=0} = 0; \quad \frac{dx_{(s)}}{dt_{(s)}}|_{t_{(s)}=0} = c.$$

If we consider the ensemble of inertons as the whole object, an inerton cloud with the rest mass m_0 , which surrounds a moving particle with the rest mass M_0 then the Lagrangian may be presented as

$$L = -M_0 c^2 \left\{ 1 - \left(\frac{1}{M_0 c^2} \right) \left[M_0 \left(\frac{dX}{dt} \right)^2 + m_0 \left(\frac{dx}{dt} \right)^2 - \frac{2\pi}{T} \sqrt{M_0 m_0} \left(X \frac{dx}{dt} + v_0 x \right) \right] \right\}^{1/2}. \quad (16)$$

Thus the particle moves along the X -axis with the velocity dX/dt (v_0 is the initial velocity); x is the distance between the inerton cloud and the particle, dx/dt is the velocity of the inertons cloud, and $1/T$ is the frequency of collisions between the particle and cloud. The equations of motion are reduced to the following

$$\frac{d^2 X}{dt^2} + \frac{\pi}{T} \frac{v_0}{c} \frac{dx}{dt} = 0; \quad (17)$$

$$\frac{d^2 x}{dt^2} - \frac{\pi}{T} \frac{c}{v_0} \left(\frac{dX}{dt} - v_0 \right) = 0. \quad (18)$$

The corresponding solutions to eqs. (17) and (18) for the particle and the inerton cloud are

$$\begin{aligned} \frac{dX}{dt} &= v_0 \cdot (1 - |\sin \frac{\pi t}{T}|); \\ X(t) &= v_0 t + \frac{\lambda}{\pi} \cdot \left\{ (-1)^{[\frac{t}{T}]} \cos \frac{\pi t}{T} - (1 + 2[\frac{t}{T}]) \right\}; \end{aligned} \quad (19)$$

$$\lambda = v_0 T;$$

$$x = \frac{\Lambda}{\pi} |\sin \frac{\pi t}{T}|;$$

$$\frac{dx}{dt} = c (-1)^{[\frac{t}{T}]} \cos \frac{\pi t}{T}; \quad (20)$$

$$\Lambda = c T.$$

Expressions (19) show that the velocity of the particle periodically oscillates and λ is the amplitude of particle's oscillations along its path. In particular, λ is the period of oscillation of the particle velocity that periodically changes between v_0 and zero. The inertons cloud periodically leaves the particle and then comes back; Λ is the amplitude of oscillations of the cloud.

The frequency of collisions of the particle with the inerton cloud allows the presentation in two ways: 1) via the collision of the particle with the cloud, i.e., $1/T = v_0/\lambda$ and 2) via the collision of the inerton cloud with the particle, i.e., $1/T = c/\Lambda$. These two expressions result into the relationship

$$\frac{v_0}{\lambda} = \frac{c}{\Lambda}, \quad (21)$$

which connects the spatial period λ of oscillations of the particle with the amplitude Λ of the inertons cloud, i.e., maximal distance to which inertons are removed from the particle.

If we introduce a new variable

$$\frac{d\mathcal{X}}{dt} = \frac{dx}{dt} - \frac{\pi}{T} X \sqrt{\frac{M_0}{m_0}} \quad (22)$$

in the Lagrangian (16), we arrive to the canonical form on variables for the particle

$$L = -M_0 c^2 \left\{ 1 - \frac{1}{M_0 c^2} \left[M_0 \left(\frac{dX}{dt} \right)^2 - M_0 \left(\frac{2\pi}{2T} \right)^2 X^2 + m_0 \left(\frac{d\mathcal{X}}{dt} \right)^2 - \frac{2\pi}{T} v_0 x \sqrt{\frac{M_0}{m_0}} \right] \right\}^{1/2}. \quad (23)$$

This Lagrangian allow us to obtain (Krasnoholovets, 1997) the effective Hamiltonian of the particle that describes its behavior relative to the center of inertia of the particle-inerton cloud system

$$H_{\text{eff}} = \frac{1}{2} \frac{p^2}{M} + \frac{1}{2} M \left(\frac{2\pi}{2T} \right)^2 X^2 \quad (24)$$

where $M = \frac{M_0}{\sqrt{1-v_0^2/c^2}}$ (and also $m = \frac{m_0}{\sqrt{1-v_0^2/c^2}}$). The harmonic oscillator Hamiltonian (24) allows one to write the Hamilton-Jacobi equation for a shortened action S_1 of the particle

$$\frac{1}{2M} \left(\frac{\partial S_1}{\partial X} \right)^2 + \frac{1}{2} M \left(\frac{2\pi}{2T} \right)^2 X^2 = E. \quad (25)$$

Here E is the energy of the moving particle. Introduction of the action-angle variables leads to the following increment of the particle action within the cyclic period $2T$ (Krasnoholovets and Ivanovsky, 1993)

$$\Delta S_1 = \oint p dX = E \cdot 2T. \quad (26)$$

One can write Eq. (26) via the frequency $\nu = 1/2T$ as well. At the same time $1/T$ is the frequency of collisions of the particle with its inertons cloud. Owing to the relation $E = \frac{1}{2} M v_0^2$ we also get

$$\Delta S_1 = M v_0 \cdot v_0 T = p_0 \lambda \quad (27)$$

where $p_0 = Mv_0$ is the particle initial momentum. Now if we equate the values ΔS_1 and Planck's constant h , we obtain instead of expressions (26) and (27) major relationships (1), which form the basis of conventional quantum mechanics.

De Broglie (1986), when writing relationships (1), noted that they resulted from the comparison of the action of a particle moving rectilinearly and uniformly (with the energy E and the momentum Mv_0) and the phase of a plane monochromatic wave extended in the same direction (with the frequency E/h and the wavelength h/Mv_0). Yet the first relation in (1) he considered as the main original axiom of quantum theory.

In our case, expressions (26) and (27) have been derived starting from the Hamiltonian (24) or the Hamilton-Jacobi eq. (25) of the particle. The main peculiarity of our model is that the Hamiltonian and the Hamilton-Jacobi equation describe a particle whose motion is not uniform but oscillatory. It is the space substrate, which induces the harmonic potential responding to the disturbance of the space by the moving particle. The oscillatory motion of the particle is characterized by the relation

$$\lambda = v_0 T \tag{28}$$

which connects the initial velocity of the particle v_0 with the spatial period of particle oscillations λ (or the free path length of the particle) and the time interval T during which the particle remains free, i.e. does not collide with its inerton cloud. On the other hand, relation (28) holds for a monochromatic plane wave that spreads in the real physical space: λ is the wavelength, T is the period and v_0 is the phase velocity of the wave. Thus with the availability of the harmonic potential, the behavior of the particle follows the behavior of a wave and, therefore, such a motion should be marked by a very specific value of the adiabatic invariant, or increment of the particle action ΔS_1 within the cyclic period. It is quite reasonable to assume that in this case the value of ΔS_1 is minimum, which is equal to Planck's constant h .

Two relationships (1) immediately allow the deduction of the Schrödinger equation (de Broglie, 1986). Moreover, the presence of the proper time of a particle in the Schrödinger equation (Krasnoholovets, 1997) signifies that the equation is Lorentz invariant. The wave ψ -function acquires a sense of the imaging of a real wave function that characterizes the motion of a complicated formation – the particle and its inerton cloud. The real wave function (and its wave ψ -function imaging or map) is defined in the range that is exemplified by the dimensions of the particle's inerton cloud: λ along the particle path and 2Λ in transversal directions. In such a manner, inertons acquire the sense of a substructure of the matter waves and should be treated as carriers of inert properties of matter. Heisenberg's uncertainty relations gain a deterministic interpretation as a quantum system now is complemented by the inerton cloud; therefore, an unknown value of the momentum of the particle automatically is compensated by the corresponding momentum of the particle's inertons.

4 Spin and relativistic approximation

The notion of spin of a particle is associated with an intrinsic particle motion. Several tens of works have been devoted to the spin problem. Major of them is reviewed in the recent author's paper (Krasnoholovets, 2000a). Here we add some recent references: Chashihin (2000), Rangelov (2001), Danilov et al. (1996), Plyuschay (1989, 1990, 2000). Main ideas of the works quoted in Krasnoholovets (2000a), and in the mentioned references are reduced to a moving particle that is surrounded by a wave, or a small massless particle, or an ensemble of small massless particles, which engage in a circular motion.

Having tried the introduction of the notion of spin in the concept presented, let us look at the situations in which the particle spin manifest itself explicitly. First, it appears as an additional member $\pm \frac{h}{2}$ to the projection onto the z-axis of the moment of momentum $\mathbf{r} \times M\mathbf{v}_0$ of a particle. Second, it introduces the correction $\pm \frac{eB_z h}{2M}$ to the energy of a charged particle in the magnetic field with the projection of the induction onto the z-axis equals to B_z . Third, it provides for the Pauli exclusion principle.

Of course, it seems quite reasonable to assume that the spin in fact reflects some kind of proper rotation of the particle. However, we should keep in mind that the operation 'rotor' is typical for the electromagnetic field that the particle generates in the environment when it starts to move. In other words, the appearance of the electromagnetic field in the particle surrounding one may associate just with its proper rotation of some sort. In our concept, superparticles that form the space net are not rigid; they fluctuate and allow local stable and unstable deformations. Thus the particle may be considered as not rigid as well. In this case along with an oscillating rectilinear motion, the particle is able to undergo some kind of an inner pulsation, like a drop. Besides the pulsation can be oriented either along the particle velocity vector or diametrically opposite to it. Then the Lagrangians (16) and (23) change to the matrix form (Krasnoholovets, 2000a)

$$\mathcal{L} = ||\mathcal{L}_\alpha||, \quad \alpha = \uparrow, \downarrow. \quad (29)$$

The function \mathcal{L}_α can be written as

$$\mathcal{L}_\alpha = -gc^2 \sqrt{1 - \frac{1}{g^2 c^2} [U_{(\text{spat})} + U_{(\text{intr})\alpha}]}. \quad (30)$$

Here $U_{(\text{spat})}$ is the same as in expression (2) and $U_{(\text{intr})\alpha}$ is similar to $U_{(\text{spat})}$, however, all spatial coordinates (and velocities) are replaced for the intrinsic ones: $\mathbf{X} \rightarrow \vec{\Xi}_\alpha$ for the particle and $\mathbf{x} \rightarrow \vec{\xi}_\alpha$ for the inerton cloud. So inertons carry bits of the particle pulsation as well. The intrinsic motion is treated as a function of the proper time of the particle t . Then the equations of motion and the solutions to them are quite similar to those obtained in the previous section. The intrinsic velocity $d\Xi_\alpha/dt$ ranges between $\pm v_0$ ("+" if $\alpha = \uparrow$ and "-" if $\alpha = \downarrow$) and zero; $d\xi_\alpha/dt$ ranges between $\pm c$ and zero within the segment 2Λ of the spatial path of the inerton cloud. Such a motion is characterized by relationships similar to (26) and (27) and hence is marked by Planck's constant h .

The intrinsic variables do not appear in the case of a free moving particle. However, an external field being superimposed on the system is able to engage into the variables. Then we can write the wave equation for the spin variable of the particle

$$\left(\frac{\hat{\Pi}_\alpha^2}{2M} - e_\alpha \varepsilon_\alpha\right) \chi_\alpha = 0 \quad (31)$$

where the operator $\hat{\Pi}_\alpha = (\hat{\pi}_\alpha - e\mathbf{A})$ and $\hat{\pi}_\alpha = -i\hbar d/d\vec{\Xi}_\alpha$ is the operator of the intrinsic momentum of the particle, e and \mathbf{A} are the electric charge and the vector potential of the field, respectively. χ_α is the eigenfunction and ε_α is the eigenvalue; the function $e_\alpha = 1$ if $\alpha = \uparrow$ and $e_\alpha = -1$ if $\alpha = \downarrow$.

If the induction of the magnetic field has only one component B_z aligned with the z -axis, the solution to eq. (31) becomes

$$\varepsilon_\alpha = e_\alpha \frac{e\hbar B_z}{2M}; \quad (32)$$

$$\chi_\alpha = \pi^{-1/4} \exp\left[-\frac{\pi_{\alpha x} - eA_x}{2e\hbar B_z}\right]. \quad (33)$$

So $\varepsilon_{\uparrow,\downarrow} = \pm \frac{e\hbar B_z}{2M}$ and therefore the eigenvalues of the so-called spin operator $\mathbf{S}_{\uparrow,\downarrow}$ are

$$S_{\uparrow,\downarrow z} = \pm \frac{\hbar}{2}; \quad S_{\uparrow,\downarrow x} = S_{\uparrow,\downarrow y} = 0. \quad (34)$$

Thus, the intrinsic motion introduced above satisfies the behavior of a particle in the magnetic field. The total orbital moment of the electron in an atom includes the spin contribution proceeding just from the interaction of the electron with an magnetic field. Moreover, the availability of two possible antipodal intrinsic motions of the particle allows the satisfaction of the Pauli exclusion principle. Consequently, the model of the spin described complies with the three said requirements.

Now the total Hamiltonian of a particle can be represented in the form of

$$H_{\uparrow,\downarrow} = c\sqrt{\mathbf{p}^2 + \vec{\pi}_{\uparrow,\downarrow}^2 + M_0^2 c^2} \quad (35)$$

(a similar Hamiltonian describes the particle's inerton cloud). As can easily be seen from expression (35), the spin introduces an additional energy to the particle Hamiltonian transforming it to a matrix form. Then following Dirac we can linearize the matrix $H_{\uparrow,\downarrow}$ and doing so we will arrive to the Dirac Hamiltonian

$$\hat{H}_{\text{Dirac}} = c\hat{\alpha}\hat{p} + \hat{\rho}_3 M_0 c^2. \quad (36)$$

At this point, information on the matrix operators $\hat{\pi}_{\uparrow,\downarrow}$ goes into the Dirac matrices. Thus from the physical point of view the Dirac transformation (36) is substantiated only in the case when the initial Hamiltonian is a matrix as well. And just this fact has been demonstrated in the theory proposed.

The Dirac formalism is correct in the range $r \geq h/Mc$ and is restricted by the amplitude of inerton cloud $\Lambda = \lambda c/v_0$. At $r < h/Mc$ the approach described above

can easily be applied. It has been pointed out (Krasnoholovets, 2000a) that the inerton cloud and the oscillatory mode of the crystallite's superparticles, which vibrate in the environment of the particle, cause the nature of spinor components. Two possible projections of spin enlarge the total number of the Dirac matrices and the spinors to four.

The submicroscopic consideration allows one sheds light on the interpretation of the so-called negative kinetic energy and the negative mass of rest of a free particle, which enter into the solutions of the corresponding Dirac equation (see, e.g. Schiff, 1959). The negative spectral eigenvalues $E_- = -\sqrt{c^2\mathbf{p}^2 + m^2c^4}$ are interpreted as states with the negative energy of the particle (and because of that Dirac proposed to refer it to the energy of the positron). However, the presence of the inerton cloud that oscillates near the particle lets us to construe the eigenvalues of the particle as a spectrum of "left" and "right" inerton waves which respectively emitted and absorbed by the particle. Such waves, $\varphi_{\alpha-} = \varphi_{\alpha}(r - at)$ and $\varphi_{\alpha+} = \varphi_{\alpha}(r + at)$ where α specifies the spin projection, depend on the space variables which can be made identical, while the time variable t is entered as either $+t$ or $-t$. In quantum mechanics the operator $i\hbar\partial/\partial t$ just corresponds to the particle energy E . Thus, we can interpret the positive eigenvalue E_+ as the total energy of the inerton cloud that moves away from the particle while the negative eigenvalue E_- as the total energy of the inerton cloud that comes back to the particle.

This must be paralleled with the recent research conducted by Dubois (2000a,b), who has studied anticipation in physical systems considering anticipation as their inner property which is embedded in the system but is not a model-based prediction. In particular, it has been shown by Dubois that such a property is inherent to electromagnetism and quantum mechanics. Namely, Dubois (2000b) started from space-time complex continuous derivatives which were constructed in such a way that it gave the discrete forward and backward derivatives $\partial^{\pm}/\partial t$. Here, Dubois's methodology may be justified in terms of the present submicroscopic approach because the derivative $\partial^+/\partial t$ might be referred to inertons flying away from the particle and the derivative $\partial^-/\partial t$ might be assigned to inertons moving backward to the particle. Besides the two types of velocities are present in anticipatory physical systems, so called "phase" and "group" velocities. These two velocities would also be ascribed to two opposite flows of inertons. We can also emphasize that the Dubois' idea about the masses of particles as properties of space-time shifts is also very close to the author's and Bounias (1990) hypothesis on mass as a local deformation in the space net. Note that the hypothesis has found future trends (Bounias and Krasnoholovets, 2002): it allows evidence in terms of the topology and fractal geometry.

5 Inertons in action: experimental verification

§1. The photoelectric effect occurring under strong irradiation in the case that the energy of the incident light is essentially smaller than the ionization potential of gas atoms and the work function of the metal has been reconsidered from the submi-

croscopic viewpoint. It has been shown (Krasnoholovets, 2001a) that the (nonlinear) multiphoton theory, which has widely been used so far, and the effective photon concept should be changed for a new methodology. The author's approach was based on the hypothesis that inerton clouds are expanded around atoms' electrons. That means that the effective cross-section σ of an atom's electron together with the electron's inerton cloud falls within the range between λ^2 and Λ^2 (i.e. $10^{-16} \text{ cm}^2 < \sigma < 10^{-12} \text{ cm}^2$) that much exceeds the cross-section area of the actual atom size, 10^{-16} cm^2 . The intensity of light in focused laser pulses used for the study of gas ionization and photoemission from metals was of the order of 10^{12} to 10^{15} W/cm^2 . Thus several tens of photons simultaneously should pierce the electron's inerton cloud and at least several of them could be engaged with the cloud's inertons and scattered by them. Consequently, the electron receives the energy needed to release from an atom or metal. The theory indeed has been successfully applied to the numerous experiments (Krasnoholovets, 2001a).

§2. In condensed media, inerton clouds of separate particles (electrons, atoms, and molecules) should overlap forming the entire elastic inerton field, which densely floods in the media. It has been theoretically shown (Krasnoholovets and Byckov, 2000) that in this case the force matrix W that determines branches of acoustic vibrations in solids comprises of two members: $W = V_{ac} + V_{iner}$. Here the first member is responsible for the usual elastic electromagnetic interaction of atoms and is responsible just for the availability of acoustic properties of solids, but the second one is originating from the overlapping of atoms' inerton clouds. It is remarkable that each of the members is involved in the expression for W equally. Therefore, an inerton wave striking an object will influence the object much as an applied ultrasound. Among the features of ultrasound, one can call destroying, polishing, and crushing. It was anticipated that inerton waves would act on specimens in a similar manner. A power source of inerton waves is the Earth: any mechanical fluctuations in the Earth should generate corresponding inerton waves. Two types of inerton flows one can set off in the terrestrial globe. The first flow is caused by the proper rotation of the Earth. Let A be a point on the Earth surface from which an inerton wave is radiated. If the inerton wave travels around the globe along the West-East line, its front will pass a distance $L_1 = 2\pi R_{\text{Earth}}$ per circle. The second flow spreads along the terrestrial diameter; such inerton waves radiated from A will come back passing distance $L_2 = 4R_{\text{Earth}}$. The ratio is

$$\frac{L_1}{L_2} = \frac{\pi}{2}. \quad (37)$$

If in point A we locate a material object which linear sizes (along the West-East line and perpendicular to the Earth surface) satisfy relation (37), we will receive a resonator of the Earth inerton waves.

We have studied specimens (razor blades) put into the resonator for several weeks. By using the scanning electron microscope, in fact, we have established difference in the fine morphological structure of cutting edge of the razor blades while the morphologically more course structure remains well preserved.

Note that the Earth inerton field is also the principal mover that launched rather fantastic quantum chemical physical processes in Egypt pyramids (Krasnoholovets, 2001b), power plants of the ancients that has recently been proved by Dunn (1998).

§3. Just recently, the inerton concept has been justified in the experiment on the searching for hydrogen atoms clustering in the δ -KIO₃·HIO₃ crystal (Krasnoholovets et al., 2001). It has been assumed that vibrating atoms should induce the inerton field within the crystal. This in turn should change the paired potential of interatomic interaction. Taking into account such a possible alteration in the potential, we have calculated the number of hydrogen atoms in a cluster and predicted its properties. Then the crystal has been investigated by using the Bruker FT IR spectrometer in the 400 to 4000 cm⁻¹ spectral range. Features observed in the spectra unambiguously have been interpreted just as clustering of hydrogen atoms.

6 Concluding remarks

Thus, we have uncovered that the interpretation of quantum mechanics in the framework of the double solution theory indeed is possible. However, the theory presented is distinguished from de Broglie's (1987), which he actively developed seeking for the solution of deterministic interpretation of the problem. The major point of the given concept is an original cellular construction of a real space, the introduction of notions of the particle, mass, and elementary excitations of the space. The mechanics constructed is based on the Lagrangians (16) and (23), equations of motion, and solutions to them, (17)-(22). The Lagrangians explicitly include elementary excitations of the space, which accompany a moving particle and directly interact with the particle. The main peculiarities of the mechanics called submicroscopic quantum (or wave) mechanics are the free path lengths for the particle λ and its inerton cloud Λ and, because of that, the mechanics is similar to the kinetics theory. The particle velocity v_0 is connected with λ by relation $v_0 = \lambda/T$ where $1/T$ is the frequency of the particle collisions with the inerton cloud (and $1/2T = \nu$ is the frequency of the particle oscillation along its path). Since the motion of the particle is of oscillating nature, it permits the construction of the Hamiltonian-Jacobi equation (25) and the obtaining the minimum increment of the particle action within the period ν^{-1} that is identified with Planck's constant h . This allows one to derive the principal quantum mechanical relations (1) and then construct the Schrödinger and Dirac formalisms.

Submicroscopic quantum mechanics has solved the spin problem reducing it to special intrinsic pulsations of a moving particle. As a result, an additional correction (positive or negative) is introduced to the particle's Hamiltonian transforming it to a matrix form that in its turn has provided the reliable background to the Dirac's linearization of the classical relativistic Hamiltonian.

Inertons are treated as a substructure of the matter waves and yet inertons surrounding moving particles are identified with carriers of inert properties of the particles. The inerton concept also determines the boundaries of employment of the wave

ψ -function and spinor formalisms reducing the boundaries to the range covered by inerton cloud amplitude Λ of the particle studied.

At last, inertons, which widely manifest themselves in numerous experiments, can be treated as a basis for anticipation in physical systems because just inertons represent those inner properties to which Dubois (2000a,b) referred constructing anticipation as actually embedded in the systems.

Further studies need widening the scope of applying of quantum mechanics. In particular, one could apply inertons to the problem of quantum gravity because inertons are shown to be real carriers of gravitational interaction. Bounias (2002) has just found other application of inertons, namely, to biological systems: the availability of the inerton wave function of an object allowed him to construct the Hamiltonian of living organism considering it as an anticipatory operator of evolution.

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