

On the theory of the anomalous photoelectric effect stemming from a substructure of matter waves

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Abstract

The two opposite concepts – multiphoton and effective photon – readily describing the photoelectric effect under strong irradiation in the case that the energy of the incident light is essentially smaller than the ionization potential of gas atoms and the work function of the metal are treated. Based on the submicroscopic construction of quantum mechanics developed in the previous papers by the author the analysis of the reasons of the two concepts discrepancies is led. Taking into account the main hypothesis of those works, i.e., that the electron is an extended object that is not point-like, the study of the interaction between the electron and a photon flux is carried out in detail. A comparison with numerous experiments is performed.

Key words: space, matter waves, inertons, laser radiation, photoelectric effect

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Quantum description of interaction of light and matter; related experiments

1 Introduction

The previous papers of the author [1-3] present a quantum theory operating at the scale $\sim 10^{-28}$ cm (this size combines all types of interactions as required by the grand unification of interaction). The theory takes into account such general directions as: deterministic view on quantum mechanics pioneered by L. de Broglie and D. Bohm (see, e.g. Refs. 4,5), the search for a physical vacuum model in the form of a real substance (see, e.g. Refs. 6-11), the introduction in the united models as if a "superparticle" whose different states are the electron, muon, quark, etc. [12], and the model of polaron in a solid, i.e., that a moving charged carrier strongly interacts with a polar medium.

The kinetics of a particle constructed in works [1-3] easily results in the Schrödinger and Dirac formalisms at the atom scale. Besides the developed theory could overcome the two main conceptual difficulties of standard nonrelativistic quantum theory. First, the theory advanced a mechanism which could naturally remove long-range action from the nonrelativistic quantum mechanics. Second, the Schrödinger equation gained in works [1,2] is Lorentz invariant owing to the invariant time entered the equation.

The main distinctive property of the theory was the prediction of special elementary excitations of space surrounding a moving particle. It was shown that space should always exhibit resistance to any canonical particle when it starts to move: the moving particle rubs against space and such a friction generates virtual excitations called "inertons" in papers [1,2]. Thus the inerton cloud around a moving particle one can identify with a volume of space \mathfrak{V} that the canonical particle excites at its motion. In other words, the inerton cloud may be considered as a substructure of the matter waves which are described by the wave ψ -function in the region of the \mathfrak{V} .

Nonetheless, the question arises whether one can reveal a cloud of inertons, which accompany a single canonical particle. As was deduced in Ref. 1, the amplitude of spatial oscillations of the inerton cloud Λ/π correlates with the amplitude of spatial oscillations of the particle, that is, the de Broglie wavelength of the particle λ :

$$\Lambda = \lambda c/v_0 \tag{1}$$

where v_0 is the initial velocity of the particle and c is the initial velocity of inertons (velocity of light). If $v_0 \ll c$ then $\Lambda \gg \lambda$ and hence the disturbance of space in the form of the inerton cloud should appear in an extensive region around the particle. In this connection, the cloud of inertons may be detected, for instance, by applying a high-intensity luminous flux.

To examine this assertion, let us turn to experimental and theoretical results available when laser-induced gas ionization phenomena and photoemission from a laser-irradiated metal take place.

2 The two opposite concepts

First reports on the experimental demonstration of the laser-induced gas ionization occurrence at a frequency below the threshold appeared in the mid-1960s (Meyerand and Haught [13], Voronov and Delone [14], Smith and Haught [15] and others). Those works launched detailed experimental and theoretical study of the new unexpected phenomena. At the time being, it would seem the mechanism providing the framework for the phenomena has been roughly understood. However, this is not the case: all materials available one can subdivide into two different classes.

Having taken a critical view of the effect in which the photon energy of the incident light is essentially smaller than the ionization potential of atoms of rarefied noble gases and the work function of the metal, we shall turn to the two opposite standpoints excellently expounded in the reviews by Agostini and Petite [16] and Panarella [17,18]. At the same time it should be particularly emphasized that any improvement of the multiphoton theory is not the aim

of the present work. The author wishes only to show that something more fundamental is hidden behind the formalism of orthodox quantum mechanics that is employed as a base for the study of a matter irradiated by an intensive light.

2.1 Multiphoton concept

The review paper by Agostini and Petite [16] analysed several tens of works exploiting the prevailing multiphoton theory. The multiphoton concept is based on the typical interaction Hamiltonian

$$\hat{H}_{\text{int}} = -e\hat{z}\vec{E}_0 \cos \omega t \quad (2)$$

which specifies the interaction between the dipole moment $e\vec{z}$ of an atom and the incident electromagnetic field $\vec{E} = \vec{E}_0 \cos \omega t$. The concept starts from the standard time dependent perturbation theory, Fermi [19], describing a probability per unit time of a transition of an atom from the bound state $|i\rangle$ to a state $|c\rangle$ in the continuum. On the next stage the concept modifies the simple photoelectric effect to the nonlinear one (see, e.g. Keldysh [20] and Reiss [21]) in which the atom is ionized by absorption of several photons. The \mathcal{N} th-order time dependent perturbation theory changes the usual Fermi golden rule to \mathcal{N} photon absorption that produces the probability [16]

$$w_{\mathcal{N}} = \frac{2\pi}{\hbar} \left(\frac{2e^2}{\varepsilon_0 c} \right)^{\mathcal{N}} \sum_c \left| \sum_{i,j,\dots,k} \frac{\langle g|z|i\rangle \langle i|z|j\rangle \dots \langle k|z|c\rangle}{(E_g + \hbar\omega - E_i)(E_g + 2\hbar\omega - E_j)\dots} \right|^2 \quad (3)$$

where $|i\rangle$, $|j\rangle$, ..., $|k\rangle$ are the atomic states, I the intensity of laser beam and $|c\rangle$ the continuum states with energy $E_g + \hbar\omega$, E_g being the energy of the ground state $|g\rangle$. The summation over intermediate states could be performed by several methods. An estimation of the probabilities of multiphoton processes can be made utilizing the so-called generalized cross section [16]

$$s_{\mathcal{N}} = 2\pi(8\pi\alpha)^{\mathcal{N}} r^{2\mathcal{N}} \omega^{-\mathcal{N}+1} \quad (4)$$

where $r \sim 0.1$ nm is the effective atom radius and $\alpha = 1/137$ is the fine structure constant. The Einstein law $E = \hbar\omega$ characterizing the simple photoelectric effect changes to the relation specifying the nonlinear photoelectric effect

$$E_c = \mathcal{N}\hbar\omega - E_i. \quad (5)$$

The \mathcal{N} -photon ionization rate (3) is proportional to $I^{\mathcal{N}}$. This prediction, as was pointed out by Agostini and Petite [16], verified experimentally up to $\mathcal{N} = 22$ and with laser intensity up to 10^{15} W/cm². They noted that "I must be below the saturation intensity to perform this measurement. When I approaches to I_s , one must make out account the depletion of the neutral atom population, which modifies the intensity dependence of the ion number". It may be seen from the preceding that $I_s \geq 10^{15}$ W/cm².

At the same time we should note that the experiment does not point clearly to the dependence of $I^{\mathcal{N}}$. The experiment only demonstrates that in a log-log plot N_i versus light intensity I where N_i is the number of ionized atoms of gas all points are located along a straight line whose slope is proportional to \mathcal{N} . This was shown by Lompre *et al.* [22] for Xe, Kr, and Ar with an accuracy about 2%. Such result was interpreted [22] as a simultaneous absorption of \mathcal{N} photons; the linear slope was held to 2×10^{13} W/cm² and the maximum value was $\mathcal{N} = 14$.

The authors of the review [16] marked that good agreement the multiphoton theory and experiment had till the experimental investigation (Martin and Mandel [23] and Boreham and Hora [24]) of the energy spectra of electrons ejected in the ionization of atoms; the kinetics energy of ejected electrons was far in excess of the prediction. Since then the multiphoton concept has advanced to so-called the above-threshold ionization (ATI). It replaced relationship (5) for

$$E_c = (\mathcal{N} + S)\hbar\omega - E_i \quad (6)$$

where S is the positive integer. Several consequences were checked experimentally: branching ratios (Petite *et al.* [25] and Kruit *et al.* [26]), the intensity dependence, i.e., proportionality to I^{N+S} (Fabre *et al.* [27], Agostini *et al.* [28] and others [16]).

An attempt to verify the nonlinear photoelectric effect on metals was undertaken by Farkas [29] (however, see below).

During the last decade a number of further studies of the multiphoton ionization of atoms under ultra intensive laser radiation have been performed both experimentally and theoretically (see, e.g. review papers and monographs [30-37]). For example, papers of Avetissian *et al.* [35,38] deal with the relativistic theory of ATI of hydrogen-like atoms; at the same time the authors note that the idea of introducing of the stimulated bremsstrahlung for the description of the photoelectron final state still remains as a great problem for the ATI process. Besides the definition of wave dynamic function of an ejected electron stands problematic as well.

Unfortunately the major deficiency of the ATI and more advanced models is too complicated expressions for the probability of ejected photoelectrons. Such expressions need additional assumptions. Hence a distinguish feature of the nonlinear multiphoton theory is the availability of a great many free parameters. Besides all the recent experiments operate with extremely short laser pulses which rather strikes atoms then slowly excite them. And this has cast some suspicion on the application of the time dependent perturbation theory (nonrelativistic or relativistic) for the description of ejection of photoelectrons from atoms in all cases. More likely femtosecond laser pulses create new effects which need new detailed studies (such as the scattering of electrons radiated from atoms immediately after ionization that tries to account the eikonal approximation [38], etc.).

Thus the results obtained with different lasers might be different as well. Below we will analyze only the pure multiphoton concept that became the starting point for the further complications; in other words we will treat the case of the adiabatic turning on electromagnetic perturbation. Notwithstanding the fact that the multiphoton methodology is widely recognize today, we should emphasize that it ignored some "subtle" experimental results obtained with the use of nanosecond and picosecond light/laser pulses in the 1960s and 1970s (perhaps setting that such results were caused by indirect reasons).

2.2 *Effective photon concept*

In review papers Panarella [17,18] analysed about a hundred of other experiments devoted a laser-induced gas ionization and laser-irradiated metal. Panarella explicitly described all dramatic events connected with the construction of a reasonable mechanism which could explain unusual experimental data on the basis of standard concepts of quantum theory. Based on those experimental results he convincingly demonstrated the inconsistency of generally accepted multiphoton methodology. In particular, Panarella studied the following series of experiments: 1) variation of the total number N_i of ionized gas atoms as a function of the laser intensity I_p (see Refs. 17,18 and also Agostini *et al.* [39]). In a log-log plot the experimental points did not lie on a straight line and the inflection point, for all gases studied, got into the range approximately from 10^{12} to 10^{13} W cm⁻² at the laser wavelength $1.06 \mu\text{m}$ and from 10^{11} to 10^{12} W/cm² at the laser wavelength $0.53 \mu\text{m}$ (note that such an inflection point, as was mentioned in the previous subsection, should refer to the saturation intensity whose value I_s , however, of the order of 10^{15} W/cm²!); 2) variation of the total number N_i as a function of time t of the increase in intensity of laser pulse (the experiment by Chalmeton and Papoular [40]); 3) variation of the breakdown intensity threshold against the gas density (see experiments by Okuda *et al.* [41-43]); 4) focal volume dependence of the breakdown threshold intensity (see, e.g. the experiment by Smith and Haught [15]); and others.

All those experiments could not be explained in the framework of the multiphoton

methodology. The multiphoton concept failed to interpret just fine details revealed in the experiments. Among other things Panarella stressed that the experiment by Chalmerton and Papoular [40] was a crucial one.

The cascade theory (see, e.g. Zel'dovich and Raizer [44]) was also untenable to explain a number of data (see Ref. 17). This theory conjectured that random free electrons with the great energy were present in the gas and those electrons along with newly formed electrons generated other electrons; it was conceived that the optical field accelerated the electrons.

Panarella analysed several other theoretical hypotheses which assumed the existence of high-than-normal energy photons in laser beam: the model based on quantum formalism, Allen [45], the model based on quantum potential theory, Dewdney *et al.* [46,47], and the model resting on classical electromagnetic wave theory of laser line broadening, de Brito and Jobs [48] and de Brito [49]. The first two models operated with the Heisenberg uncertainty principle and de Broglie-Bohm quantum potential respectively; it was expected that the deficient energy of a photon could appear due to some quantum effects. The last model suggested that the existence of separate high-energy photons in the laser beam might be stipulated by the laser line shape. Unfortunately the models could not explain the whole series of available experimental results.

In contrast to those concepts, Panarella noted [17] that new physics should be present in the phenomena described above and proposed an effective photon theory [17,18]. He postulated that the photon energy expression $\varepsilon = h\nu$ had to be modified "ad hoc" into the novel one:

$$\varepsilon = \frac{h\nu}{1 - \beta_\nu f(I)} \quad (7)$$

where $f(I)$ is the function of the light intensity and β_ν is a coefficient. In this manner Panarella's theory holds that, at the extremely high intensities of light, photon-photon interaction begins to play a significant role in the light beam such that the photon energy becomes a function of the photon flux intensity. To develop an effective photon concept it was pointed out [18] that the number density of photons in the focal volume is much larger than $\tilde{\lambda}^{-3}$ where $\tilde{\lambda}$ is the wavelength of laser's irradiated light. In this respect he came up with the proposal to reduce the photon wavelength in the focal volume. He assumed that it unquestionably followed from quantum electrodynamics that photons could not come any closer than $\tilde{\lambda}$.

The effective photon concept satisfied all available experimental facts mentioned above in this subsection. Moreover the concept was successfully applied to Panarella's own first-class experiments on electron emission from a laser irradiated metal surface [50,51,18] and to other experiments (Pheps [52] and see also Refs. 17,18).

Such remarkable success of the formula (7) gave rise to the confidence that some hidden reasons could be a building block for understanding the principles of effective photons formation [18]. An elementary consideration of photons and hence effective photons based on neutrinos has been constructed by Raychaudhuri [53].

Thus in this section we have given an objective account of facts and adduced the two absolutely opposite views on the same phenomena. So we need to establish the reasons for the main discrepancies between the multiphoton and effective photon concepts and then develop an approach that would reconcile them.

3 Interaction between the photon flux and an electron's in- erton cloud

First of all we need to discuss in short such notions as the photon and photon flux. On question, what is photon?, quantum electrodynamics answers (see, e.g. Berestetskii *et al.*

[54]): it is something that can be described by the equation

$$\partial^2 \vec{A} / \partial t^2 - c^{-2} \partial^2 \vec{A} / \partial \vec{r}^2 = 0 \quad (8)$$

where \vec{A} is the vector potential that satisfies the condition

$$\text{div} \vec{A} = 0. \quad (9)$$

The vector potential operator $\hat{\vec{A}}$ of the free electromagnetic field is constructed in such a way that each wave with a wavevector \vec{q} corresponds to one photon with the energy $h\nu_{\vec{q}}$ in the volume V , that is, $\hat{\vec{A}}$ is normalized to V in accordance with the formula (see, e.g. Davydov [55,56])

$$\hat{\vec{A}}(\vec{r}, t) = \sum_{\vec{q}, \alpha} \left(\frac{ch}{|\vec{q}|V} \right)^{1/2} e^{i\vec{q}\vec{r}} \vec{j}_\alpha(\vec{q}) \left(\hat{a}_{\vec{q}\alpha}(t) + \hat{a}_{-\vec{q}\alpha}^\dagger(t) \right) \quad (10)$$

where c is the velocity of light, h is Planck's constant, \vec{q} is the wave vector ($|\vec{q}| = 2\pi/\tilde{\lambda}$), $\vec{j}_\alpha(\vec{q})$ is the unit vector of the α th polarization, $\hat{a}_{\vec{q}\alpha}^\dagger$ ($\hat{a}_{\vec{q}\alpha}$) is the Bose operator of creation (annihilation) of a photon, and V is the volume containing the electromagnetic field.

A pure particle formalism can also be applied to the description of the free electromagnetic field; in this case each of the particles – photons – has the energy $\varepsilon = h\nu$ and the momentum $\hbar\vec{k}(= h\nu/c)$. Just such "photon language" is often more convenient. It admits to consider a monochromatic electromagnetic field as a single mode which contains a number of photons.

Now let us start by considering the origin of the disagreements between the two opposite concepts.

That was considerable success of the multiphoton concept that it incorporated \mathcal{N} photons whose total energy was equal to the potential of ionization of an atom, expression (5). A prerequisite for the construction of the concept was the supposition that there was strong nonlinear interaction between a laser beam and a gas.

Criticism: The multiphoton methodology does not take into account the threshold light intensity needed for gas ionization. The photoelectric effect, as such, is not investigated, the methodology only suggests that atoms of gas may be excited to the energy level (5) in the continuum. Besides the methodology ignores the fact of the coherence of the electromagnetic field irradiated by laser. At the same time the problem of electromagnetic radiation may be reduced to the problem of totality of harmonic oscillators, ter Haar [57], which in the case of the laser radiation must be regarded as coherent. This means that each of the \mathcal{N} photons absorbed should have the same right, but using the \mathcal{N} th-order time dependent perturbation theory one adds photons successively. (The distinction between the incoherent and coherent electromagnetic field is akin to that between the normal and superconducting state of the same metal in some sense. Indeed in a superconductor electrons can not be considered separately: all superconducting phenomena are caused by cooperate quantum properties of electrons. That is why describing superconducting phenomena one should include the cooperation of electrons, for instance the Meissner-Ochsenfeld effect.)

The advantages of the effective photon concept are its flexibility at the analysis of experimental results. The concept assumed the existence of the threshold light intensity that launches ionization of atoms of gas and ejection of electrons from the metal. The effective photon was deduced from the assumption that there could not more than one orthodox photon in a volume of space $\sim \tilde{\lambda}^3$. Owing to the huge photon density in the laser pulse the concept conjectured that photons could interact with each other forming "effective photons" (7). The latter are absorbed as the whole and the absorption is a linear process, which is highly similar to the simple photoelectric effect.

Criticism: Photons are subjected to Bose-Einstein statistics and this means that it is not impossible that the volume V contains an enormous number of photons with the same

energy $h\nu_{\vec{q}_0}$. In other words, the density of photons depends on the initial conditions of the electromagnetic field generation. In any event the statistics is absolutely true at the atom (and even nucleus) scale, i.e., so long as the photon concentration in the pulse does not far exceed the concentration of atoms in a solid $\sim 10^{23} \text{ cm}^{-3}$. (Note that a somewhat similar pattern is observed when the intensity of sound in a crystal is enhanced. In the original state acoustic phonons obey the Planck distribution, but when the ultrasound is switched on, the phonon density increases while the volume of the crystal remains the same.)

Having described ionization of atoms of gas and photoemission from a metal in terms of the submicroscopic approach [1-3], an effort can be made to try to develop a theory of the anomalous photoelectric effect in which electron's wide spread inerton cloud simultaneously absorbs a number of coherent photons from the intensive laser pulse. Thus the theory will combine Panarella's idea on the anomalous photoelectric effect and the idea of the multiphoton concept on simultaneous absorption of \mathcal{N} photons.

We shall assume that in the first approximation atoms of gas and the metal may be considered as systems of quasi-free electrons. The Fermi velocity of s and p electrons in an atom is equal to $(1-2) \times 10^8 \text{ cm/s}$. Setting $v_F = v_0 \simeq 2 \times 10^8 \text{ cm/s}$ one obtains $\lambda = h/mv_0 \simeq 0.36 \text{ nm}$ (m is the electron mass) and then in accordance with relation (1) the amplitude of oscillations of the inerton cloud equals $\Lambda/\pi \simeq 17 \text{ nm}$. The cloud has anisotropic properties: it is extended on λ along the electron path, that is, along the velocity vector \vec{v}_0 , and on $2\Lambda/\pi$ in the transversal directions. This means that the cross section σ of the electron together with its inerton cloud in the systems under consideration should satisfy the inequalities:

$$\frac{\lambda^2}{4\pi} < \sigma < \frac{\Lambda^2}{\pi}, \quad \text{or} \quad 10^{-16} \text{ cm}^2 < \sigma < 1.7 \times 10^{-12} \text{ cm}^2; \quad (11)$$

here one takes into account that the radius of electron's inerton cloud equals Λ/π . At the same time the cross-section of an atom is only $\sim 10^{-16} \text{ cm}^2$. The intensity of light in (10-100)-psec focused laser pulses used for the study of gas ionization and photoemission from metals was of the order of $10^{12} - 10^{15} \text{ W/cm}^2$, that is, $10^{30} - 10^{33} \text{ photons/cm}^2$ per second. Dividing this intensity into the velocity of light one obtains the concentration of photons in the focal volume $n \simeq 3 \times (10^{19} - 10^{22}) \text{ cm}^{-3}$ and hence the mean distance between photons is $n^{-1/3} \simeq (30 - 3) \text{ nm}$. The number of photons bombarding the inerton cloud around an individual electron is $\sigma n^{2/3}$; this value can be estimated, in view of inequality (11), as

$$\begin{aligned} 1 < \sigma n^{2/3} < 10^3 & \text{ at } n \approx 3 \times 10^{19} \text{ cm}^{-3} & \text{ and} \\ 1 < \sigma n^{2/3} < 10^5 & \text{ at } n \approx 3 \times 10^{22} \text{ cm}^{-3}. \end{aligned} \quad (12)$$

The next thing to do is to write the model interaction between the electron inerton cloud and an incident coherent light. In an ordinary classical representation the electron in the applied electromagnetic field is characterized by the energy

$$\mathcal{E} = \frac{1}{2m} (\vec{p} - e\vec{A})^2 \quad (13)$$

where \vec{A} is the vector potential of the electromagnetic field. This usually implies that the vector potential \vec{A} in Ampère's formula (13) relates to the field of one photon. This is confirmed by expression (11) and the supposition that the electron can be considered as a point in its classical trajectory. In the language of quantum theory this means that both the wave function of the electron and the wave function of the photon are normalized to one particle in the same volume V , Berestetskii *et al.* [58]. However, as follows from the analysis above, the electron jointly with its inerton cloud is an extended object. Because of this, it can interact with many photons simultaneously and the coupling function between the electron and the applied coherent electromagnetic field should be defined by the density of the photon flux. Therefore, contrary to the usual practice to use the approximation of

single electron-photon coupling (13) in all cases, one can introduce the approximation of the strong electron-photon coupling

$$\mathcal{E} = \frac{1}{2m}(\vec{p} - e\vec{A}_{\text{eff}})^2 \quad (14)$$

which should be correct in the case of simultaneous absorption/scattering of \mathcal{N} photons by the electron. Thus in (13)

$$\vec{A}_{\text{eff}} = e\vec{A}\mathcal{N}, \quad \mathcal{N} = \sigma n^{2/3}. \quad (15)$$

In experiments involving noble gases discussed by Panarella [17,18] the laser pulse intensity had the triangular shape. We shall apply the same approach. In other words, let the intensity be changed over the duration Δt of the laser pulse whose intensity runs along the two equal sides of the isosceles triangle, that is from $I = 0$ at $t = 0$ to the peak intensity $I = I_p$ at $t = t_p = \Delta t/2$ and then to $I = 0$ at $t = \Delta t$.

Thus \vec{A}_{eff} becomes time dependent; it can be present in the form

$$\vec{A}_{\text{eff}}(\vec{r}, t) = \vec{A}_p e^{i\vec{k}\vec{r} - i\omega t} \mathcal{N}(t) \quad (16)$$

where \vec{A}_p is the vector potential of the electromagnetic field at the peak intensity of the pulse,

$$\mathcal{N}(t) = \sigma_{\text{th}} n_{\text{th}}^{2/3} \frac{t}{t_p} \quad (17)$$

is the number of photons absorbed by the electron where $n_{\text{th}}^{2/3}$ is the effective photon density in the unit area at the threshold intensity of the laser pulse when the energy of $\sigma_{\text{th}} n_{\text{th}}^{2/3}$ photons reaches the absolute value of the ionization potential of atoms or the work function of the metal, that is $h\nu\sigma_{\text{th}} n_{\text{th}}^{2/3} = W$. As relation (17) indicates the cross section of electron's interton cloud, σ is also signed by dependence on the threshold intensity; much probably σ is not constant and depends on the velocity of the electron, the frequency of incident light and the light intensity. The presentation (16) is correct within the time interval $\Delta t/2$, that is, $t \in [0, t_p]$.

Hence passing on to the Hamiltonian operator of the electron in the intensive field one has

$$\hat{H} = \frac{\hat{p}^2}{2m} - \frac{e}{m} \vec{A}_{\text{eff}}(r, t) \hat{p}; \quad (18)$$

here we are restricted to the linear field effect, much as it is made in the theory of simple photoelectric effect (see, e.g. Berestetskii *et al.* [58], Blokhintsev [59], and Davydov [60]).

In the case of the simple photoelectric effect the Schödinger equation for the electron

$$i\hbar \frac{\partial \psi}{\partial t} = (\hat{\mathcal{H}} + \hat{\mathcal{W}}(\vec{r}, t))\psi \quad (19)$$

contains the Hamiltonian operator $\hat{\mathcal{H}}$ of the electron in an atom (or the metal) and the interaction operator

$$\hat{\mathcal{W}}(\vec{r}, t) = -\frac{e}{m} \vec{A}(\vec{r}, t) \hat{p} \quad (20)$$

whose matrix elements are much smaller than those of the operator $\hat{\mathcal{H}}$. However in our case the matrix elements of the operator

$$\hat{\mathcal{W}}_{\text{eff}}(\vec{r}, t) = -\frac{e}{m} \vec{A}_{\text{eff}}(\vec{r}, t) \hat{p} \quad (21)$$

do not seem to be small due to the great value of \vec{A}_p . Therefore, exploiting the perturbation theory, we should resort to the procedure, which makes it feasible to extract a small parameter.

Nonetheless, the necessary smallness is already inserted into the structure of the vector potential $\vec{A}_{\text{eff}}(\vec{r}, t)$: the number of photons absorbed by the electron is a linear function of the duration of the growing intensity of the pulse [see (17)]. Consequently the interaction operator (21) can be safely used for $t \ll t_p$.

4 Anomalous photoelectric effect

In the absence of the external field the Schrödinger equation

$$i\hbar \frac{\partial \psi_0}{\partial t} = \hat{\mathcal{H}} \psi_0 \quad (22)$$

which describes the electron (in an atom or metal) has the solution

$$\psi_0 = e^{-i\frac{\hat{\mathcal{H}}}{\hbar}t}. \quad (23)$$

Eq. (22) is transformed in the presence of the field to the equation

$$i\hbar \frac{\partial \psi}{\partial t} = (\hat{\mathcal{H}} + \hat{\mathcal{W}}_{\text{eff}}(t))\psi. \quad (24)$$

The ψ function from (24) can be represented in the form (see, e.g. Fermi [19])

$$\psi(\vec{r}, t) = e^{-\frac{\hat{\mathcal{H}}}{\hbar}t} \sum_l a_l(t) \psi_l(\vec{r}); \quad (25)$$

here $a_l(t)$ are coefficients at eigenfunctions $\psi_l(\vec{r})$. By substituting function (25) into Eq. (24) and multiplying a new equation by $\psi_f^*(\vec{r})$ to left and then integrating over \vec{r} one obtains

$$i\hbar \frac{\partial a_f(t)}{\partial t} = \sum_l a_l(t) \langle f | \hat{\mathcal{W}}_{\text{eff}} | l \rangle e^{i\omega_{fl}t} \quad (26)$$

where $\hbar\omega_{fl} = E_f - E_l$, $E_{f(l)}$ is the eigenvalue of Eq. (22) and the matrix element

$$\langle f | \hat{\mathcal{W}}_{\text{eff}} | l \rangle = -\frac{e}{m} \int \psi_f^* \vec{A}_{\text{eff}}(\vec{r}, t) \hat{\vec{p}} \psi_l d\vec{r}. \quad (27)$$

In the first approximation the coefficient is equal

$$a_f^{(1)} \simeq \frac{1}{i\hbar} \int_0^t \langle f | \hat{\mathcal{W}}_{\text{eff}} | l \rangle e^{i\omega_{fl}\tau} d\tau. \quad (28)$$

The possibility of the transition from the atomic state E_l to the state of ionized atom E_f (or the possibility of the ejection of electron out of the metal) is given by the expression

$$P(t) \equiv P_f(t) = |a_f^{(1)}(t)|^2, \quad (29)$$

or in the explicit form

$$P(t) = \left| \frac{1}{i\hbar} \frac{e}{m} \langle f | \vec{A}_p \hat{\vec{p}} | l \rangle \right|^2 \left| \int_0^t \mathcal{N}(\tau) e^{i(\omega_{fl} - \omega)\tau} d\tau \right|^2. \quad (30)$$

The first factor in (30) is well known in the simple photoelectric effect, because it defines the probability of the electron transition from the atomic $|l\rangle$ to the free state $\langle f|$. This

factor can be designated as $|M|^2$ and extracted from (30) in the explicit form (see, e.g. Blokhintsev [59]):

$$\begin{aligned} |M|^2 &\equiv \left| \frac{1}{i\hbar} \langle f | \vec{A}_p \vec{p} | l \rangle \right|^2 \\ &= 16\pi \frac{e^2 \hbar^2}{m^2 V} \vec{A}_p^2 \left(\frac{Z}{a_{\text{Bohr}}} \right)^5 \left(\frac{\hbar}{\vec{p}_{\text{free}}} \right)^6 \frac{\sin^2 \theta \cos^2 \phi}{(1 - \frac{v}{c} \cos \theta)^4}; \end{aligned} \quad (31)$$

here V is a normalizing volume, a_{Bohr} is Bohr's radius, Z is the number charge, \vec{p}_{free} is the momentum of the stripped electron. The last factor in (31) shows that the momentum \vec{p}_{free} falls within the solid angle $d\Omega$ (v is the velocity of the free electron and $|\vec{p}_{\text{free}}| = mv$). Taking into account that the vector potential \vec{A} of the electromagnetic field is connected with the intensity I of the field through the formulas

$$I = \varepsilon_0 c^2 |\vec{E}|^2, \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} = i\omega \vec{A}_p e^{i(\omega t - \vec{k}\vec{r})} \quad (32)$$

we gain the relation

$$\vec{A}_p^2 = \frac{1}{\varepsilon_0 c^2 \omega^2} I_p. \quad (33)$$

The intensity I_p can be separated out of the matrix element (31), i.e., we can write

$$|M|^2 = |\mathcal{M}|^2 I_p \quad (34)$$

where

$$|\mathcal{M}|^2 = 16\pi \frac{e^2 \hbar^2}{\varepsilon_0 c^2 \omega^2 m^2 V} \vec{A}_p^2 \left(\frac{Z}{a_{\text{Bohr}}} \right)^5 \left(\frac{\hbar}{\vec{p}_{\text{free}}} \right)^6 \frac{\sin^2 \theta \cos^2 \phi}{(1 - \frac{v}{c} \cos \theta)^4}; \quad (35)$$

Now, expression (30) can be rewritten as

$$P(t) = |\mathcal{M}|^2 I_p |\mathcal{I}(t)|^2 \quad (36)$$

where

$$|\mathcal{I}(t)|^2 = \int_0^t \mathcal{N}^*(\tau) e^{-i(\omega_{fl} - \omega)\tau} d\tau \int_0^t \mathcal{N}(\tau) e^{i(\omega_{fl} - \omega)\tau} d\tau. \quad (37)$$

Let us calculate the integral $\mathcal{I}(t)$:

$$\begin{aligned} \mathcal{I}(t) &= \int_0^t \mathcal{N}(\tau) e^{i(\omega_{fl} - \omega)\tau} d\tau = \frac{\sigma_{\text{th}} n_{\text{th}}^{2/3}}{t_p} \int_0^t \tau e^{i(\omega_{fl} - \omega)\tau} d\tau \\ &= \frac{\sigma_{\text{th}} n_{\text{th}}^{2/3}}{t_p} \left[\frac{t}{i(\omega_{fl} - \omega)} e^{i(\omega_{fl} - \omega)t} + \frac{1}{(\omega_{fl} - \omega)^2} (e^{i(\omega_{fl} - \omega)t} - 1) \right]. \end{aligned} \quad (38)$$

Substituting $\mathcal{I}(t)$ and $\mathcal{I}^*(t)$ into (37) one obtains

$$\begin{aligned} |\mathcal{I}(t)|^2 &= \frac{(\sigma_{\text{th}} n_{\text{th}}^{2/3})^2}{t_p^2 (\omega_{fl} - \omega)^2} \left\{ t^2 - \frac{2t}{(\omega_{fl} - \omega)} \sin((\omega_{fl} - \omega)t) \right. \\ &\quad \left. + \frac{2}{(\omega_{fl} - \omega)^2} [1 - \cos((\omega_{fl} - \omega)t)] \right\}. \end{aligned} \quad (39)$$

In our case $\omega_{fl} - \omega = (E_f - E_l)/\hbar - \omega$ where $\omega = 2\pi\nu$ and ν is the frequency of incident light. As $\omega_{fl} \gg \omega$, one can put $\omega_{fl} - \omega \simeq \omega_{fl}$. Besides we consider the approximation when $t \ll t_p = \Delta t/2 \approx 10^{-8} - 10^{-7}$ s. Hence for the wide range of time (i.e., $\omega_{fl}^{-1} \ll t \ll t_p$ the inequality $\omega_{fl} t \gg 1$ is held and expression (39) can be replaced by

$$|\mathcal{I}(t)|^2 \simeq \left(\frac{\sigma_{\text{th}} n_{\text{th}}^{2/3}}{\omega_{fl} t_p} \right)^2 t^2. \quad (40)$$

The matrix element ω_{fl} in (40) can be eliminated by substituting the absolute value of ionization potential of atoms (or the work function of the metal) W , that is, $\omega_{fl} \rightarrow W/\hbar$. If we substitute (40) into (36), we finally get

$$\begin{aligned} P(t) &= |\mathcal{M}|^2 \left(\frac{\hbar \sigma_{\text{th}} n_{\text{th}}^{2/3}}{W t_{\text{p}}} \right)^2 I_{\text{p}} t^2 \\ &\equiv |\mathcal{M}|^2 (\hbar \mathcal{N} / W t_{\text{p}})^2 I_{\text{p}} t^2. \end{aligned} \quad (41)$$

In the case when the incident laser pulse one may consider as a perturbation that is not time dependent, the interaction operator (21) can be regarded as a constant value

$$\begin{aligned} \hat{W}_{\text{eff}} &= -\frac{e}{m} \vec{A}_{\text{eff}}(\vec{r}) \hat{\vec{p}} \\ &\equiv -\frac{e}{m} \mathcal{N} \vec{A}_0 e^{i\vec{k}\vec{r}} \hat{\vec{p}} \end{aligned} \quad (42)$$

between the moments of cut-in and cut-off and $\hat{W}_{\text{eff}} = 0$ behind the time interval Δt corresponding to the duration of the laser pulse. Now having the interaction operator (42) we directly use the Fermi golden rule and obtain the probability of the anomalous photoelectric effect (compare with the theory of the simple photoelectric effect, e.g. Refs. 59, 60)

$$P_0 = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \mathcal{N}^2 I V m |\vec{p}_{\text{free}}| \Delta t d\Omega; \quad (43)$$

here $|\mathcal{M}|^2$ is the matrix element defined above (35), $\mathcal{N} = \sigma n^{2/3}$ is the number of photons absorbed by an atom (or the metal) simultaneously, I is the typical intensity of the laser pulse, $V m |\vec{p}_{\text{free}}|$ is the density of states (V is the normalizing volume, m is the electron mass and $|\vec{p}_{\text{free}}|$ is the momentum of the stripped electron).

Thus it is easily seen that the interaction between the laser pulse and gas atoms (or the metal) is not nonlinear. This is why the results to be expected from this new approach would correlate with the results predicted by the effective photon (7).

5 Discussion

Let us apply the results obtained above to the experimental data used by Panarella [17,18] for the verification of the effective photon. Besides other experimental results are taken into account as well. We shall restrict our consideration to qualitative evaluations, which note only the general tendency towards the behaviour of the systems in question.

5.1 Laser-induced gas ionization

5.1.1. Let probability (41) describes the transition from the stationary state of an atom to the ionized state of the same atom. Multiplying both sides of expression (41) by the concentration N_a of gas atoms which are found in the focal volume investigated, one gains the formula for the concentration N_i of ionized atoms

$$N_i = N_a |\mathcal{M}|^2 \left(\frac{\hbar \mathcal{N}_{\text{th}}}{W t_{\text{p}}} \right)^2 I_{\text{p}} t^2. \quad (44)$$

So, it is readily seen that

$$N_i \propto N_a I_{\text{p}} t^2, \quad (45)$$

that is, the concentration N_i of ionized atoms is directly proportional to the peak laser pulse intensity I_{p} and the time to the second power. Time dependence of ionization before breakdown was analysed by Panarella [17,18] in the framework of the same formula (45)

obtained by him using the effective photon. The experiment by Chalmeton and Papoular [40] showed that the evolution of free electrons knocked out of gas atoms, that is $d \ln N_e(t)/dt$, is only a function of time. As the electron density $N_e(t) = N_i(t)$, following Refs. 17,18 we obtain from (45) (or (44)): $d \ln N_e(t)/dt = 2/t$, in agreement with the experiment.

5.1.2. Temporal dependence of the breakdown threshold intensity was studied by Panarella [18] with the aid of the same expression (45). At breakdown $N_i = \text{const}$, I_p is replaced by the threshold intensity I_{th} and a time interval t is equal to the breakdown time t_b . Hence in this case expression (45) gives $I_{th} = \text{const} \times t_b^{-2}$ or, according to formulas (32), $E_{th} \propto t_b^{-1}$. This expression agrees with the experiment by Pheps [52].

5.1.3. The experimental results on the number of ions created by the laser pulse as a function of the pulse intensity can also be described in terms of the anomalous photoelectric effect. For this purpose we should concentrate upon expression (43), which yields after multiplying both sides by the concentration N_a of gas atoms

$$N_i = \text{const} \times N_a \mathcal{N}^2 I. \quad (46)$$

However before proceeding to the verification of the theory we should call attention to the process, which is the reverse of the photoelectric effect. The case in point is the radiation recombination of an electron with a fixed ion, Berestetskii *et al.* [61].

The intensity I of the laser pulse characterizes the density of electromagnetic energy per unit of time, that is, one can deem that I is in inverse proportion to time. This enables the construction of a possible model describing the occupancy of states of ions and atoms in the presence of the strong laser irradiation. The processes of ionization of atoms and recombination of ions may be represented by the following kinetic equations:

$$\dot{N}_a = \alpha N_a - \beta N_i + D; \quad (47)$$

$$\dot{N}_i = \gamma N_i - \alpha N_a \quad (48)$$

where the dot over $N_{a(i)}$ means derivation with respect to the "time" variable $\tilde{t} \equiv 1/I$. Here αN_a and βN_i present the rate of ionization and restoration of atoms of gas respectively, γN_i represents the rate of recombination of ions in gas, and D is the rate of irreversible decay of the atoms (it specifies a part of electrons which leave the gas studied). As the first approximation we can put $D = 0$ and therefore $\gamma = \beta$. Such an approximation allows the following solution of Eqs. (47) and (48):

$$N_a = N_{a0}(1 - e^{-(\alpha+\beta)/I}); \quad (49)$$

$$N_i = N_{a0} \left(\frac{\alpha}{\beta} - \frac{\alpha}{\alpha + 2\beta} e^{-(\alpha+\beta)/I} \right) \quad (50)$$

where N_{a0} is the initial concentration of atoms of gas in the focal volume. Denote the parameter $(\alpha + \beta)$ by I_m , which may correspond to an intensity supporting the balance between ionization and recombination in the gas system studied. Then substituting N_a from the solution (49) into relation (46) we get the resultant expression governs the total number of ions N_i as a function of the laser intensity I and the number of absorbed photons \mathcal{N} :

$$N_i = \text{const} \times N_{a0} \mathcal{N}^2 I (1 - e^{-I_m/I}). \quad (51)$$

Expression (51) correlates in outline with Panarella's [17,18] expression which he utilized to explain the total number of ions produced by the laser pulse (the experimental results by Agostini *et al.* [39]). In fact when $I < I_m$, the exponential term can be neglected in (51) and in a log-log plot the number of ions versus the pulse intensity is proportional to the number of absorbed photons, that is

$$\log N_i / \log I \propto \mathcal{N} \quad (52)$$

and, hence, N_i against I is a straight line whose slope is \mathcal{N} (see, e.g. the experimental results by Lompre *et al.* [22]). When $I > I_m$, the exponent can not be neglected and, therefore, a curve N_i versus I must show an inflection point (probably at $I \simeq I_m$) in accord with the experimental results by Agostini *et al.* [39].

5.1.4. Expression (51) is able to explain the breakdown intensity threshold measured as a function of pressure or gas density. If expression (51) is written in the form

$$I_{\text{th}} \simeq \text{const} \times N_{a0}^{-1} (1 + e^{-I_m/I}) \quad (53)$$

where I_{th} is the breakdown threshold intensity, the function I_{th} versus N_{a0} indicates that $I_{\text{th}} \propto N_{a0}^{-1+\delta}$ where the value δ satisfies the inequalities $0 < \delta < 1/2$. Such a variation of the parameter δ rhymes satisfactory with the experimental results by Okuda *et al.* [41-43] and their analysis carried out by Panarella [17,18].

5.1.5. The appearance of electrons released from atoms of gas at high energies (more than 100 eV at the laser intensity at 5×10^{14} W/cm², Agostini and Petite [16]) follows immediately from the theory constructed. The two possibilities may be realized. First of all expressions (41) and (43) allow the kinetic energy of revealed electrons larger than $h\nu\sigma_{\text{th}}n_{\text{th}}^{2/3}$ because as is evident from inequalities (12), an electron's inerton cloud can absorb in principle more photons, $\mathcal{N} + \Delta\mathcal{N} = \sigma n^{2/3}$, than is required for overcoming the threshold value $\mathcal{N} = \sigma_{\text{th}}n_{\text{th}}^{2/3}$. This is no surprise, since the anomalous photoelectric effect is a generalization for the simple one. In the theory of the simple photoelectric effect one can recognize the approximations $h\nu \geq W$ and $h\nu \gg W$. The first inequality can be related to the anomalous photoelectric effect considered above. The second one corresponds to the Born (adiabatic) approximation, Berestetskii *et al.* [61], and in the case of the anomalous photoelectric effect the inequality changes merely to $(\mathcal{N} + \Delta\mathcal{N})h\nu \gg W$. Notice that this inequality is in agreement with formula (6) utilized by the multiphoton theory to account for the energy spectrum of electrons ejected in the ionization of atoms.

At the same time the absorption of radiation by an accelerated electron (called the above-threshold ionization in Ref. 16) must not be ruled out. Actually, if a final state of a released electron is the state of a free electron in an electromagnetic field (so called "Volkov state" [16]), one may assume that the electron was stripped having a very small kinetic energy. Let initial velocity v_0 of the electron released from an atom be several times less than the velocity of the electron in the atom which we set equal to the Fermi velocity $v_F \simeq 2 \times 10^6$ m/s in Section 3. In such the case as it follows from relation (1) and inequalities (11) the electron excites surrounding space significantly wider than the Fermi electron and this is why the cross section of the excited range of space around the low speed electron should be at least ten times greater than the magnitude of cross section evaluated in Section 3. This means that our low speed electron will be immediately scattered by more than $\mathcal{N} + 10$ photons of the laser beam and therefore its kinetic energy may reach the value of several tens eV.

5.2 Electron emission from a laser-irradiated metal

The investigation of the photoelectric emission from a laser-irradiated metal performed experimentally by Panarella [50,51,18] has shown that:

1) the photoelectric current i_e is linear with light intensity I ,

$$i_e \propto I; \quad (54)$$

2) the maximum energy ε_{max} of the emitted electron is a function of light intensity I ,

$$\varepsilon_{\text{max}} \propto f(I) \quad (55)$$

and ε_{max} increases with I . The same dependence of i_e and ε_{max} on I is predicted by the effective photon theory [18] (note that the multiphoton methodology predicted that i_e depends on I to the power \mathcal{N} and ε_{max} depends on ν only of the light).

Let us compare the results of the anomalous photoelectric effect theory developed above with the experimental results by Panarella (formulas (54) and (55)). In his experiments the light intensity I changed from $\sim 10^6$ W/cm² to $\sim 10^9$ W/cm² from experiment to experiment. This value of I is not very great and we can take into consideration the total power transferred during one pulse. By this is meant that the light intensity is assumed to be constant during the pulse. Therefore the expression (43)

$$P_0 = \text{const} \times (\sigma n^{2/3})^2 I \quad (56)$$

can be used to evaluate of the electron emission from the metal. Expression (56) was obtained utilizing the perturbation theory. In other words, the interaction energy $\mathcal{W}_{\text{eff}} \equiv e\vec{A}_0\vec{p}\sigma n^{2/3}/m$ that forms the perturbation operator (42) should be smaller than the absolute value of the work function W . In Panarella's experiments the value of W was about 10^{-18} J (i.e., approximately 6 eV). At $I = 10^6 - 10^9$ W/cm² (i.e., $10^{24} - 10^{27}$ photons/cm² per second) one has

$$\mathcal{W}_{\text{eff}} = (1.8 \times 10^{-22} - 5.7 \times 10^{-21}) \times (\sigma n^{2/3})^2 \text{ [J]}. \quad (57)$$

If we try formally to estimate an additional number of photons $\sigma n^{2/3} - 1$ which pass their energy on to the electron that absorbed a single photon, we will find with regard for the inequality (11):

$$1 < \sigma n^{2/3} \ll 1 \quad \text{at} \quad n \approx 3 \times 10^{15} \text{ cm}^{-3}; \quad (58a)$$

$$1 < \sigma n^{2/3} < 2 \quad \text{at} \quad n \approx 10^{18} \text{ cm}^{-3}. \quad (58b)$$

Substituting $\sigma n^{2/3}$ from (58b) in expression (57), it is easily seen that the inequality $W \gg \mathcal{W}_{\text{eff}}$ is not broken, that is formula (56) could be applied to the study of anomalous electron emission from the metal. Nonetheless, inequalities (58a) are not correct while the experiment [51,18] pointed to the presence of photoelectrons at the light intensity $I = 10^6$ W/cm² ($n \approx 3 \times 10^{15}$ cm⁻³). One way around this problem is to take into account the large concentration of electrons n_{elec} in a metal. Indeed, the value of $n_{\text{elec}} \sim 10^{21}$ cm⁻³ and consequently the mean distance between electrons is $n_{\text{elec}}^{-1/3} \sim 1$ nm. Bearing in mind that owing to relationship (1) the electron's inerton cloud in the metal is characterized by amplitude $\Lambda/\pi \simeq 17$ nm, one should supplement the parameter σ by a correlation function $F(\Lambda, n_{\text{elec}}^{1/3})$. The function can be chosen in the form

$$F = \left[\frac{\Lambda}{n_{\text{elec}}^{-1/3}} \right]^\gamma, \quad \gamma > 0. \quad (59)$$

The function (59) corrects inequalities (58a). Hence expression (56) takes the form

$$P_0 = \text{const} \times (\sigma n^{2/3} F)^2 \times I \quad (60)$$

and it can be used until $W \gg \mathcal{W}_{\text{eff}}$. For large I_p when $\mathcal{W}_{\text{eff}} \sim W$, expression (60) is also suitable, but only at the initial stage of the laser pulse (in this case the factor t/t_p should again be introduced into the right hand side (60)). Note that in the case of rarefied gases the overlapping of inerton clouds of neighboring atoms begins for their concentration $n_{\text{atom}} \geq 10^{17}$ cm⁻³; here the mean distance between atoms $n_{\text{atom}}^{-1/3} \sim 20$ nm.

Comparing expressions (54) and (60) we notice that they agree: expression (60) describes the probability of the appearance of free electrons and hence their current i_e at the difference of electric potential as a linear function of I .

The behaviour of emitted electrons described by expression (55) is consistent with the prediction of the present theory as well. Panarella [51] pointed out that the incident laser beam did not heat the metal specimen. This statement is correct for the background temperature, i.e. phonon temperature of the small specimen. However the electron temperature should increase with the intensity of light; it is well known phenomenon called heat electrons (see, e.g. Refs. 62-64). The greater the light flux intensity, the greater the kinetic energy

of the heat electrons in small metal specimens [63,64]. As a result the work function W of the specimen becomes a function of the intensity of light I : W falls as I increases. Thus, expression (55) should also follow from the theory based on the inerton concept; the theory gives the explicit form of expression (55):

$$\varepsilon_{\max} = \mathcal{N}h\nu - W(I) \quad (61)$$

where $h\nu$ is the photon energy of incident light, \mathcal{N} is the threshold number of photons scattered by the electron's inerton cloud and $W(I)$ is the work function depending on the intensity of light I .

6 Conclusion

The present theory of anomalous photoelectric effect has been successfully applied to the numerous experiments where the photon energy of incident light is essentially smaller than the ionization potential of gas atoms and the work function of the metal. This theory is based on submicroscopic quantum mechanics developed in the previous papers by the author [1-3]. Note that ideas on the microstructure of the space set forth in that author's research are in excellent agreement with the recent construction of a mathematical space carried out by Bounias and Bonaly [65] and Bounias [66]. Space reveals its properties through the engagement of the particle with it. As a result – a cloud of inertons, that is, elementary excitations of the space, is created in the surrounding of the particle and just these clouds enclosing electrons were detected in the experiments mentioned above by a high-intensity luminous flux.

It is obvious that clouds of inertons, which accompany electrons were fixed also in another series of experiments carried out by a large group of physicists, Briner *et al.* [67]. Their article is entitled "Looking at Electronic Wave Functions on Metal Surfaces" and it contains the colored spherical and elliptical figures, which the authors called "the images of ψ wave functions of electrons". However, the wave ψ -function is only a mathematical function that sets connections between parameters of the system studied. So the wave ψ -function can not be observed in principle. This means that the researchers could register perturbations of space surrounding the electrons in the metal, i.e., clouds of inertons accompanying moving electrons. It is believed that mobile small deformations of space – inertons, which constitute a substructure of the matter waves – promise new interesting effects and phenomena [68,69].

At the same time, for the description of a whole series of phenomenological aspects of effects caused by highly intensive laser radiation in the case when the adiabatic approximation may be used, Panarella's effective photon theory [17,18] is also suitable (the theory is similar to the phenomenological theory of propagation of electromagnetic waves in nonlinear media, see, e.g. Ref. 70). As it follows from the analysis above, the effective photon methodology, indeed, specifies the effective photon density, or the number of photons absorbed by the electron's inerton cloud (see expression (15)); therefore, the methodology allows the correct calculation of the photon energy absorbed by an atom of gas or an electron in the metal and, as the rule, just the value of this energy is very significant for the majority of the problems which are researched.

As for the nonlinear multiphoton concept, its basis should be altered to the linear one, that is, to the anomalous photoelectric concept developed herein.

An important conclusion arising from the theory considered in the present work is that the Ampère's formula $\vec{p} - e\vec{A}$ is not universal. In the general case, when the intensity of the electromagnetic field is high, it should be replaced by the formula $\vec{p} - e\mathcal{N}\vec{A}$ where the vector potential \vec{A} is normalized to one photon and \mathcal{N} is the quantity of coherent photons scattering/absorbing by the electron's inerton cloud simultaneously. In other words, for highly intensive electromagnetic field, one should use the approximation of the strong electron-photon coupling (see expressions (14) and (15)).

The submicroscopic approach is not only advantageous in the study of matter under strong laser irradiation. The approach provides a means of more sophisticated analysis of the nature of matter waves and the nature of light. Thereby such an analysis is able to originate radically new view-points on the structure of real space, the notions of particle and field and their interaction.

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